1 Introduction

While nearly 90% of all school-aged children in the U.S. attend government-owned schools, more and more parents are dissatisfied with the perceived quality of their children’s public schools. Overall, urban schools are apparently failing to educate a large proportion of their inner-city students.

As Poterba (1994) has pointed out, one of the standard justifications of government intervention in private markets is redistribution: education, it is argued, is a fundamental right which should not be allocated according to ability to pay. But if parental resources are unequal, a well functioning private market for education will result in differences in the quality of education that children receive. These differences in education imply differences in earning opportunities, which may be seen as unfair since they are beyond the child’s control.

For historical reasons, most education services in the US are provided locally and funded by property taxes. Since communities differ in their tax bases and willingness to pay, levels of spending on education differ greatly across communities and redistribution is limited. Consequently, the level of educational achievement depends on family income. Rich parents can choose between public and private schools for their children, and the community in which they reside. In contrast, poor parents must send their children to the nearest public school, whether or not this is educationally effective or even safe.

Let us take the example of Rhode Island. There are 36 school districts in the state of Rhode Island and two state-run schools. Due to space considerations, we show only a few districts: The city of Providence, two wealthy communities (Barrington and East Greenwich), and two poorer communities (North Providence and East Providence). Tables I, II, III, and IV present the relevant information. Table I shows differences in median income, income per capita, tax base and tax rates across communities. Note that richer communities (in terms of median income or income per capita) have larger tax bases and lower tax rates.

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Table I: The Resources

<table>
<thead>
<tr>
<th>District</th>
<th>Median Family Income ($)</th>
<th>Per capita Income ($)</th>
<th>Property Value Per Pupil ($)</th>
<th>Property Tax Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhode Island</td>
<td>39,172</td>
<td>14,981</td>
<td>309,319</td>
<td>–</td>
</tr>
<tr>
<td>Providence</td>
<td>28,342</td>
<td>11,838</td>
<td>144,233</td>
<td>30.42</td>
</tr>
<tr>
<td>Barrington</td>
<td>59,483</td>
<td>24,965</td>
<td>565,523</td>
<td>20.00</td>
</tr>
<tr>
<td>East Greenwich</td>
<td>61,843</td>
<td>26,163</td>
<td>603,171</td>
<td>22.15</td>
</tr>
<tr>
<td>East Providence</td>
<td>37,634</td>
<td>14,387</td>
<td>316,927</td>
<td>35.75</td>
</tr>
<tr>
<td>North Providence</td>
<td>39,556</td>
<td>16,569</td>
<td>343,701</td>
<td>27.95</td>
</tr>
</tbody>
</table>


Table II presents some information about the students in the different communities. Note that (i) richer communities have smaller student populations; (ii) there is no big difference in the proportion of students enrolled in public and private schools; (iii) there are sizeable differences in the proportions of white and minority students across communities: in Providence, 75% of the students in public schools belong to a minority group, in East Greenwich the share is 4% and in Barrington it is 2%.

Table II: The Students

<table>
<thead>
<tr>
<th>District</th>
<th>Total Enrollment (%)</th>
<th>Public Total</th>
<th>Minority</th>
<th>White</th>
<th>Private</th>
<th>Other</th>
<th>Other Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhode Island</td>
<td>148,977</td>
<td>85.35</td>
<td>21</td>
<td>79</td>
<td>14.37</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Providence</td>
<td>29,197</td>
<td>82.44</td>
<td>75</td>
<td>25</td>
<td>17.36</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Barrington</td>
<td>3,363</td>
<td>85.61</td>
<td>2</td>
<td>98</td>
<td>13.97</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>East Greenwich</td>
<td>2,691</td>
<td>83.39</td>
<td>4</td>
<td>96</td>
<td>16.57</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>East Providence</td>
<td>8,091</td>
<td>84.32</td>
<td>14</td>
<td>86</td>
<td>15.49</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>North Providence</td>
<td>4,438</td>
<td>80.47</td>
<td>10</td>
<td>90</td>
<td>19.33</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

Source: RIPS

Table III is particularly interesting. First, Barrington and East Greenwich finance education mainly out of local tax revenues, while the other communities use both local and state funds. Federal funds play a minor role in all cases. Secondly, there is no big difference in total expenditure per pupil. However, there are some differences in expenditure devoted to general instruction (as opposed to administrative support, non-instructional services to students, transportation, management of facilities, etc.): richer communities spend more.
### Table III: Revenues and Expenditures

<table>
<thead>
<tr>
<th>District</th>
<th>Revenues (%)</th>
<th>Expenditures ($1,000)</th>
<th>Total Per Pupil ($)</th>
<th>G. Instruction ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local</td>
<td>State</td>
<td>Federal</td>
<td>Total</td>
</tr>
<tr>
<td>Rhode Island</td>
<td>57.12</td>
<td>39.07</td>
<td>3.81</td>
<td>1,046,302,424</td>
</tr>
<tr>
<td>Providence</td>
<td>36.24</td>
<td>57.55</td>
<td>6.21</td>
<td>168,833,352</td>
</tr>
<tr>
<td>Barrington</td>
<td>88.53</td>
<td>9.4</td>
<td>2.07</td>
<td>17,237,507</td>
</tr>
<tr>
<td>East Greenwich</td>
<td>91.39</td>
<td>7.42</td>
<td>1.19</td>
<td>15,560,127</td>
</tr>
<tr>
<td>East Providence</td>
<td>58.20</td>
<td>38.87</td>
<td>2.93</td>
<td>43,910,377</td>
</tr>
<tr>
<td>North Providence</td>
<td>65.49</td>
<td>32.80</td>
<td>1.71</td>
<td>26,983,501</td>
</tr>
</tbody>
</table>

Source: RIPS

Table IV describes how students leaving the system perform. In richer communities graduation rates are much higher and dropout rates much lower. A higher proportion of seniors take the S.A.T. and their average scores are higher.

### Table IV: The Results

<table>
<thead>
<tr>
<th>District</th>
<th>Graduation Rate %</th>
<th>Dropout Rate %</th>
<th>Seniors taking %</th>
<th>Math Score</th>
<th>Verbal Score</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhode Island</td>
<td>82.7</td>
<td>17.3</td>
<td>64</td>
<td>489</td>
<td>495</td>
<td>984</td>
</tr>
<tr>
<td>Providence</td>
<td>75.2</td>
<td>24.8</td>
<td>64</td>
<td>438</td>
<td>442</td>
<td>880</td>
</tr>
<tr>
<td>Barrington</td>
<td>95.2</td>
<td>4.8</td>
<td>97</td>
<td>533</td>
<td>522</td>
<td>1055</td>
</tr>
<tr>
<td>East Greenwich</td>
<td>92.3</td>
<td>7.7</td>
<td>96</td>
<td>536</td>
<td>539</td>
<td>1075</td>
</tr>
<tr>
<td>East Providence</td>
<td>77.7</td>
<td>22.3</td>
<td>59</td>
<td>464</td>
<td>472</td>
<td>936</td>
</tr>
<tr>
<td>North Providence</td>
<td>90.9</td>
<td>9.1</td>
<td>44</td>
<td>481</td>
<td>494</td>
<td>975</td>
</tr>
</tbody>
</table>

Data from RIPS.

It should be clear from the above tables that “other things equal” any reasonable parent living in Providence would rather live in Barrington or East Greenwich, where expenditure per pupil is similar to that in Providence but where the quality of education, as shown in Table IV, is much higher. If some residents of Providence use its public education system, there must be some “moving costs” (e.g., zoning regulations, transportation costs to work, or amenities), which render the education offered by the other cities too costly. It seems that the present situation is not efficient: a parent residing in Providence could be given the local per pupil expenditure and be allowed to use it to finance his child’s education in Barrington, without relocating there. This would constitute a Pareto improvement.

This fact has not been overlooked by many frustrated parents who are looking at possible alternatives to the U.S. educational system. Some proposed reforms involve one form or another of school vouchers, all of which involve disbursing public money to help parents pay for the cost of sending their children to the school of their choice, typically a school other than the district public school. It has been argued
that school vouchers will improve education by inducing competition among schools to attract students. With guaranteed enrollment rates and a low fraction of parents with the necessary means to opt out of the system, public schools have little incentive to provide good education. Friedman (1962) argues that vouchers may result in a more efficient use of public resources in education.

However, there are also equity implications to vouchers. School vouchers could exacerbate economic, racial or ability segregation. If the voucher is less than needed to cover tuition in a private school, only relatively rich parents could use them, leaving the poorest children in the public schools, with perhaps even less resources. In addition, private schools are not obliged to accept all applicants—they may accept only the smart ones. This would have an adverse effect on the students staying in the public school system if one believes that peer effects are an important determinant of the quality of education.\footnote{Equity issues are not the only arguments cited by voucher opponents. Many think that sending children to religious schools using public money violates the constitutional separation of Church and State. Moreover, some private schools oppose vouchers on the grounds that such funding will imply more regulation of the private sector. Others are afraid that the equalization of education quality across districts may lead to an equalization of housing prices, losing capitalization rents.}

Although several voucher experiments are currently underway, e.g., in Minnesota, Wisconsin, New York, and Florida, empirical evidence is still scarce. While Hoxby (1994) suggests that private school competition would greatly increase public-school effectiveness, other studies disagree (e.g., Figlio and Stone (1997)).

A theoretical approach to the subject has proven difficult. Ideally, one would like an integrated framework of community choice, housing and schooling, with agents differing by a variety of characteristics including, in addition to income, ability and race. Moreover, the framework should be appropriate to study the efficiency and equity effects of vouchers discussed above. Needless to say, most models abstract from some relevant issues and focus on some others. In many cases, the theoretical literature relies on computational methods to produce examples or to calibrate models to the available data. One strand of literature assumes that the quality of public education is homogeneous across communities and analyzes the effect of school vouchers that allow children to attend private schools. Without trying to be exhaustive, we might mention Ireland (1990) and Hoyt and Lee (1998), Manski (1992) and Epple and Romano (1998). A second strand of literature works in a multi-community framework and again analyzes the effects of school vouchers that can be used to attend private schools (for example Rangazas (1995), Nechyba (1996) and Nechyba (1999)). In this paper we analyze the effects of school vouchers that can be used in other public schools in a framework where the quality of education may differ across public schools located in different communities.\footnote{Florida has recently approved a voucher system that pays for students to attend higher-rated public schools and not only private schools.}

The aim of the paper is to show, within a two-community model, that a simple voucher system can result in a Pareto improvement as an equilibrium phenomenon. By this we mean that the voucher system will result in an equilibrium outcome that Pareto dominates the equilibrium outcome in the absence of vouchers. Since we want to compare equilibria with and without vouchers, we choose to work with a very simple model similar in spirit to that of Westhoff (1977) and Fernandez and Rogerson (1996). There is a continuum of agents, characterized by income level, who must be allocated into one of two communities. Each community is characterized by a proportional income tax rate which is used to finance locally provided public education. Education is produced by means of a decreasing returns to scale production function.
and the tax rate is determined by majority vote within each community. An additional feature of our model is that one community, say Barrington, imposes a fixed, exogenous cost on its residents. This cost may represent the fact that the other city, Providence, offers more amenities, or that people work in Providence and hence there is a transportation cost involved in living in Barrington.\footnote{Fernandez and Rogerson (1997) introduce zoning restrictions in a model with perfectly elastic housing supply. These restrictions play a similar role to our fixed cost. Rangazas (1995) also considers zoning restrictions.} In equilibrium, agents are unwilling to move from one city to another. As is typical in this kind of models, equilibria will be stratified and education quality will be higher in the rich community. Further, as a result of the exogenous cost, under reasonable assumptions on the agents’ preferences and education technology, the rich community will charge a lower tax rate. Some examples in Fernandez and Rogerson (1997) show this high quality–low tax pattern, but this paper shows some sufficient conditions for it to hold.

After calculating several examples of equilibrium, we consider the introduction of a voucher system whereby the residents of the poorer community can get the local per-pupil expenditure on education and use it to finance their children’s education in the richer community, without having to relocate. Since the resulting flow of students imposes a negative externality on the absorbing community, it is required that the incoming students pay a sufficiently high price for education to maintain its original quality. In other words, the level of education of the rich community should remain unaffected after allowing the other community’s children access to the local school. We calculate several equilibria under this voucher system and show that some of them result in a Pareto improvement with respect to the equilibrium without vouchers. In these equilibria, education quality increases in both communities, and some people move from the rich community to the poor one. This last fact is consistent with the findings of Fernandez and Rogerson (1996), who suggest that “policies whose net effect is to increase the number of residents in the poorest community will tend to be Pareto improving.” We find, however, that the difference between communities in their quality of education widens, in contrast with the findings of Rangazas (1995) who, in a model with private alternatives, shows an opposite effect.

The rest of the paper is organized as follows. Section 2 introduces the benchmark model without vouchers and presents two examples, one with logarithmic preferences and the other with quasilinear preferences. Section 3 introduces vouchers and shows how they can result in a Pareto improvement. Section 4 concludes.

2 The Economy

2.1 The benchmark model

The economy has two goods: a private good (money) and a locally provided public good (education). There is a continuum of agents, represented by the closed unit interval $[0,1]$ denoted by $I$, which is endowed with the Lebesgue measurable sets and the Lebesgue measure $\lambda$ on them. Individuals have identical utility functions:

$$u(x, q),$$

(2.1)

where $x$ is the quantity of the private good and $q$ is the quality of the education consumed. Individuals differ in their initial endowment of the private good, which is represented by the Lebesgue integrable
function

\[ y : I \rightarrow \mathbb{R}_+. \]  

(2.2)

The quantity \( y(i) \) represents the amount of money owned by agent \( i \). The distribution of the initial endowments of the private good is represented by the cumulative distribution function, \( F : \mathbb{R}_+ \rightarrow [0,1] \) defined by

\[ F(y_0) = \lambda[y^{-1}([0,y_0])], \]

(2.3)

where \( y^{-1} \) is the inverse of \( y \) in equation (2.2). \( F(y) \) represents the fraction of agents with initial endowment equal to or lower than \( y \).

There are two cities: Providence (\( P \)) and Barrington (\( B \)). Living in each city entails a fixed cost \( c_J \), where \( J = P, B \). We assume that \( c_P = 0 \) and \( c_B = c > 0 \). The index \( J \) will sometimes denote the name of a city and sometimes the set of that city’s residents. This should cause no confusion.

Education is produced in each city using the private good as an input, and according to the production function, \( g \),

\[ g : \mathbb{R}_+ \rightarrow \mathbb{R}. \]

(2.4)

If \( E_J \) is the amount of input devoted to education in city \( J \), then \( g(E_J) \) is the total amount of education produced. The quality of education received by an agent living in that city is

\[ \frac{g(E_J)}{\lambda(J)}, \]

(2.5)

where \( \lambda(J) \) is the population measure of city \( J \). Note that if \( g \) is the identity function, then the quality of education is simply the average expenditure on education in the community, which is the typical specification in the literature. As long as \( g \) exhibits decreasing returns, there will be some crowding effect.\(^4\) One may think of this crowding effect as resulting from the fact that the higher the number of pupils, the more teachers need to be hired. And since teachers are of heterogeneous quality, it is harder to get good teachers as the number of students goes up. We will discuss throughout the paper how results may differ when working under a crowding or non-crowding assumption.

The primitives of the model are then the preferences, the income distribution, the education technology and the cost of living in Barrington. Therefore a typical economy will be denoted by \( \mathcal{E} = (u, y, g, c) \).

After describing the economy, we can turn to its organization. Cities are restricted to finance the provision of education via proportional income taxation. Therefore, for city \( J \) with population measure \( \lambda(J) \) and tax rate \( t \), the quality of education is

\[ q(t, J) = \frac{g \left( t \int_J y(i) d\lambda \right)}{\lambda(J)}. \]

(2.6)

Equation (2.6) is the \textit{budget constraint} of city \( J \).

\textbf{Definition 2.1} A \textit{community} \( (J, t_J, q_J) \) is a measurable subset \( J \subseteq I \) of positive measure, an income tax

\(^4\)Needless to say, this is not the only way to model crowding.
rate \( t_J \in [0, 1] \), and a quality of education, \( q_J \), such that

\[ q_J = q(t_J, J) = \frac{g(t_J \int_J y(i) d\lambda)}{\lambda(J)}, \quad (2.7) \]

i.e., \( q_J \) satisfies the budget constraint. We say that community \((J, t_J, q_J)\) is viable if for all \( j \in J \), \((1 - t_J)y(j) - c_J \geq 0\).

In this model, we require cities not only to finance public education via a proportional income tax, but also to decide on the level of taxation by majority vote.

**Definition 2.2** Let \((J, t_J, q_J)\) be a viable community. Let \( t \in [0, 1] \) be a tax rate and let \( q(t, J) \) be the education quality associated with it according to the budget constraint. Let the inhabitants of \( J \) who prefer tax rate \( t \) to \( t_J \) be denoted by \( J(t) \):

\[ J(t) = \{ i \in J : u[(1 - t)y(i) - c_J, q(t, J)] > u[(1 - t_J)y(i) - c_J, q_J] \}. \quad (2.8) \]

We say that \((J, t_J, q_J)\) is in a majority voting equilibrium if

\[ \lambda(J(t)) \leq \frac{1}{2} \lambda(J), \quad \forall t \in [0, 1]. \quad (2.9) \]

We are now ready to define the equilibrium concept that we will use in the first part of the paper.

**Definition 2.3** An equilibrium of the economy \( E = \langle u, y, g, c \rangle \) is a partition \((P, t_P, q_P), (B, t_B, q_B)\) of \( I \) into two viable communities that satisfies:

1. **Majority voting.** Both \((P, t_P, q_P)\) and \((B, t_B, q_B)\) are in a majority voting equilibrium.
2. **Free mobility.** No agent prefers the other community to his own:

\[ u[(1 - t_P)y(i), q_P] \geq u[(1 - t_B)y(i) - c, q_B], \quad \forall i \in P; \]

\[ u[(1 - t_B)y(i) - c, q_B] \geq u[(1 - t_P)y(i), q_P], \quad \forall i \in B. \quad (2.10) \]

We define an equilibrium as a partition into two communities — thus not allowing for an equilibrium with only one community — for analytical convenience. It can be checked however, that when the production function of education satisfies \( \lim_{x \to 0} g'(x) = \infty \), the quality of education becomes arbitrarily large as the community becomes small, as long as tax collection is nonnegative. This implies that any situation with only one community would be unstable.

### 2.2 Some useful propositions

We now present some general facts that will help us calculate equilibria later.

**Proposition 2.1** (Stratification)

Let \( E \) be an economy and let \((P, t_P, q_P), (B, t_B, q_B)\) be an equilibrium of \( E \). Define

\[ v(y) = u[(1 - t_P)y, q_P] - u[(1 - t_B)y - c, q_B], \quad (2.11) \]
as the utility differential between living in Providence and living in Barrington for an agent with initial endowment of the private good $y$.

There is income stratification — the poor live in Providence and the rich live in Barrington — if

$$\frac{dv(y)}{dy} < 0, \quad \forall y \in [y_l, y_h].$$

There is income stratification — the poor live in Barrington and the rich live in Providence — if

$$\frac{dv(y)}{dy} > 0, \quad \forall y \in [y_l, y_h].$$

Proof: Suppose that there exists an agent $i$ who prefers living in Barrington to living in Providence and let $\tilde{y}$ be his initial endowment. For this agent, $v(\tilde{y}) \leq 0$. If (2.12) is satisfied, $v(y) < 0$, $\forall y > \tilde{y}$, which means that all agents richer than $i$ prefer living in Barrington. Similarly, if $i$ prefers Providence to Barrington and if (2.12) holds, all individuals who are poorer than $i$ also prefer to live in Providence. A similar argument can be used when equation (2.13) holds.

Proposition 2.2 (Boundary Indifference)

Let $E = \langle u, y, g, c \rangle$ be an economy where $u$ is a continuous function and assume that the range of $y$ is a convex set. Let $\langle (P, t_P, q_P), (B, t_B, q_B) \rangle$ be a stratified equilibrium of $E$. Then there is an individual $i$ who is indifferent between the two equilibrium communities.

Proof: Assume without loss of generality that $y(p) \leq y(b)$ for all $p \in P$ and for all $b \in B$. Define the following income levels: $y_P = \sup\{y(p) : p \in P\}$ and $y_B = \inf\{y(b) : b \in B\}$. Since $B \neq \emptyset$ and $y \geq 0$, $y_B$ is well-defined. Since $P \neq \emptyset$ and there is income stratification, $y_p$ is also well-defined; moreover, $y_P \leq y_B$.

We first prove that $y_P = y_B$. If $y^* \in (y_P, y_B)$, then, since the range of $y$ is convex, there must be an agent $i$ with $y(i) = y^*$. But this agent does not belong to either of the two communities, contradicting the fact that these communities form a partition of the set of agents. Let $y^* = y_P = y_B$ and let $i$ be an agent with $y(i) = y^*$. We claim that $i$ is indifferent between $P$ and $B$. For $u[(1 - t_P)y^*, q_P] > u[(1 - t_B)y^* - c, q_B]$, we could find a sequence $(y_n)_{n \in N}$ of income levels in $\{y(b) : b \in B\}$ that converges to $y_B = y^*$ and such that $u[(1 - t_P)y_n, q_P] \leq u[(1 - t_B)y_n - c, q_B]$, for all $n$. Then, by continuity of $u$ we would have $u[(1 - t_P)y^*, q_P] \leq u[(1 - t_B)y^* - c, q_B]$, which is a contradiction. An analogous argument shows that $i$ cannot prefer $B$ over $P$.

Proposition 2.3 (Single Peaked Preferences)

Let $u(x, q)$ be a strictly concave, continuous function with $u_{12} \geq 0$. Let $g(z)$ be a concave and continuous production function, with $g' > 0$ and $g'' \leq 0$. Then preferences are single peaked on $t$.

Proof: Let $\mu(J) = \int_J y(i) d\lambda$. An agent who lives in community $J$ has the following indirect utility function:

$$V = u \left( (1 - t)y - c_J, g(t\mu(J)) \right).$$

Since $u$ and $g$ are continuous functions and $[0, 1]$ is a compact set, $V$ has a maximum there. It is enough, then, to show that $V$ is a strictly concave function of $t$, as in this case there is a unique maximizer which
is the single peak. It can be shown that

\[
\frac{d^2V}{dt^2} = y^2u_{11} - 2y\frac{\mu(J)}{\lambda(J)} g'u_{12} + \left(\frac{\mu(J)}{\lambda(J)}\right)^2 (g')^2u_{22} + \frac{\mu(J)^2}{\lambda(J)} u_{22}''
\]  

(2.15)

which, given our assumptions, is negative and therefore \( V \) is strictly concave. \( \square \)

**Proposition 2.4 (Monotonicity)**

Let \((J, t_J, q_J)\) be a community where \(t_J\) is the most preferred tax rate of the individual with median income. If \(\frac{u_2}{u_1}\) is a monotonic function of \(y\), then \((J, t_J, q_J)\) is in a majority voting equilibrium.

**Proof:** Note that \(\frac{u_2}{u_1}\) is simply the slope of the indifference curve of an agent with initial endowment \(y\) in the \((q, t)\) space. If \((q_J, t_J)\) is the most preferred quality-tax pair for agent \(i\), then the assumption that the slope is monotonic in income implies that for each tax rate \(t\), either the individuals who are richer than him or the individuals who are poorer than him prefer \(t_J\) to \(t\). When \(i\) is the individual with median income, this implies that the community is in a majority voting equilibrium. \( \square \)

It is worth pointing out that Proposition 2.1, which guarantees stratification in our model, is different from the “single-crossing” property used in Westhoff (1977) and in Fernandez and Rogerson (1996), which guaranteed stratification there. Their assumption required the slope of the indifference curve of an agent in the \((q, t)\) space to be increasing in initial income. We do not require this, as we will see in the two examples that follow, where this slope is constant or decreasing in income. Therefore, our model generates stratification in some cases that are not covered by the above models. Nor do we require the additional Assumption 2 in Fernandez and Rogerson (1997). Note also that Proposition 2.4, which guarantees that the median voter is the agent with median income, only requires this slope to be monotonic.

### 2.3 Examples

We now present two examples that illustrate the model introduced in the previous section and the kind of solutions one can get with this simple setup.

#### 2.3.1 Example 1

Let \(E_1\) be an economy where the common utility function is \(u(x, q) = \ln(x) + \ln(q)\), and let the production function of education be \(g(z) = z^\alpha, \ 0 < \alpha < 1\). Note that \(g\) exhibits decreasing returns. Further assume that \(y\) is a continuous function with range \([y_l, y_h] \subseteq \mathbb{R}_+\).

We introduce some propositions that will help us characterize and calculate the equilibrium values for this particular example.

**Proposition 2.5**

1. Preferences on \(t\) are single peaked.
2. In equilibrium, the median voter of each community is the agent with median income.
3. In every equilibrium of \(E_1\), there is income stratification, the rich live in Barrington and the poor live in Providence.
4. In every equilibrium there is an individual who is indifferent between the two communities.
Proof:

1. Follows from Proposition 2.3.

2. Since preferences are single peaked, by the median voter theorem the majority voting equilibrium in each community is given by the median voter. It can be shown that the slope of the indifference curves in the \((q, t)\) space is

\[
\frac{u_2}{yu_1} = \frac{1 - t}{q} - \frac{c}{qy}.
\]

(2.16)

Since this slope is a monotonic function of \(y\), by Proposition 2.4, the median voter is the agent with median income.

3. Let \(v(y)\) denote the utility differential between living in Providence and living in Barrington for an agent with endowment \(y\),

\[
v(y) = \ln[(1 - t_P)y] + \ln(q_P) - \ln[(1 - t_B)y - c] - \ln(q_B).
\]

(2.17)

By Proposition 2.1 it is enough to show that \(\frac{dv(y)}{dy} < 0\). And indeed,

\[
\frac{dv(y)}{dy(i)} < 0 \iff \frac{1 - t_P}{(1 - t_P)y} - \frac{1 - t_B}{(1 - t_B)y - c} < 0
\]

\[
\iff -(1 - t_P)c < 0,
\]

(2.18)

which is true since \(c > 0\) and given the utility function, in equilibrium we must have \(t_P < 1\).

4. Follows from Proposition 2.2 and the fact that the range of \(y\) is convex.

\[\square\]

The following proposition shows that in this class of economies, the rich community imposes a lower tax rate while providing a better quality of education. Some examples in Fernandez and Rogerson (1997) show this pattern but some others do not. Within the class of economies under consideration, this pattern is a general result.

**Proposition 2.6** In every equilibrium,

1. \(t_B < t_P\). The tax rate in Barrington is lower than the tax rate in Providence.

2. \(q_B > q_P\). The quality of public education in Barrington is higher than the quality of public education in Providence.

Proof:

1. Let \((P, t_P, q_P), (B, t_B, q_B)\) be an equilibrium. Consider the community of Barrington \((B, t_B, q_B)\).

By Proposition 2.5, \(t_B\) must be the preferred tax rate of the agent with median income in Barrington. Letting \(y^*_B\) be the median voter’s income, his preferences over tax rates are represented by

\[
\ln[(1 - t)y^*_B] + \ln \left( \frac{(t \int y(i)d\lambda)^\alpha}{\lambda(B)} \right).
\]

(2.19)
Consequently, his preferred tax rate is given by

\[ t_B = \frac{\alpha}{1 + \alpha} \left( 1 - \frac{c}{y_B^m} \right). \]  

(2.20)

Similarly, since \((P, t_P, q_P)\) is a majority voting equilibrium of Providence, \(t_P\) must be the preferred tax rate of the agent with median income in Providence.

This optimal tax is given by

\[ t_P = \frac{\alpha}{1 + \alpha}. \]  

(2.21)

Since \(c > 0\) and \(y_B^m > 0\), we conclude that \(t_B < t_P\).

2. By Proposition 2.5, there is an individual who is indifferent between living in Providence and living in Barrington. Denote his endowment by \(y_b = y(i_b)\).

\[
\ln[(1 - t_P)y_b] + \ln(q_P) = \ln[(1 - t_B)y_b - c] + \ln(q_B) \\
\Longleftrightarrow \quad \frac{q_B}{q_P} = \frac{(1 - t_P)y_b}{(1 - t_B)y_b - c}.
\]

Substituting for \(t_P\) and \(t_B\) from equations (2.21) and (2.20), we obtain:

\[
\frac{q_B}{q_P} = \frac{y_b}{y_b + \alpha c y_B^m - \alpha c - c},
\]  

(2.22)

which is bigger than 1 since, by Proposition 2.5, \(\frac{y_B^m}{y_B^m} < 1\). Therefore \(q_B > q_P\).

We can now use Proposition 2.5 to derive a system of equations that determines an equilibrium of the economy. The first equation identifies the agent, \(i_b\), who is indifferent between the two communities:

\[
u[(1 - t_P)y_b, q_P] = \nu[(1 - t_B)y_b - c, q_B],
\]

where \(y_b\) is his initial endowment. Using stratification, the next two equations determine the median income in Providence and in Barrington, denoted by \(y_P^m\) and \(y_B^m\) respectively.

\[
\frac{1}{2} = \frac{\int_{y_P^m}^{y_B^m} dF}{\int_{y_P}^{y_B} dF} \quad \text{and} \quad \frac{1}{2} = \frac{\int_{y_P^m}^{y_B^m} dF}{\int_{y_P}^{y_B} dF}.
\]

Since preferences are single peaked, the tax rate is the one preferred by the median voter, who (by Proposition 2.5) is the agent with median income in that community.

\[
t_P = \arg \max_t u \left[ (1 - t)y_P^m, \frac{g \left( t \int_{y_P}^{y_P^m} y dF \right)}{\int_{y_P}^{y_P^m} dF} \right],
\]

\[
t_B = \arg \max_t u \left[ (1 - t)y_B^m - c, \frac{g \left( t \int_{y_P}^{y_B^m} y dF \right)}{\int_{y_P}^{y_B^m} dF} \right].
\]
In each community, the budget constraints must be satisfied, which gives the quality of education:

\[ q_P = \frac{g(t_P \int_{y_b}^{y} ydF)}{\int_{y_b}^{y} dF}, \]
\[ q_B = \frac{g(t_B \int_{y_b}^{y} ydF)}{\int_{y_b}^{y} dF}. \]

In order to find an equilibrium, we now have to solve the former system of seven equations with seven unknowns \((y_b, y_P^m, y_B^m, t_P, t_B, q_P, q_B)^5\).

We calculate the equilibrium when \(\alpha = 1/2\) and \(i \in [0, 1]\), \(y = 9i + 1\) (or equivalently \(y \sim U[1, 10]\)).

We find the equilibrium for different values of \(c\), the fixed cost of living in Barrington. The results are summarized in Table 1.

**Table 1: Equilibrium Values with the Logarithmic Utility Function**

<table>
<thead>
<tr>
<th>(c)</th>
<th>(i_b)</th>
<th>(y_b)</th>
<th>(t_B)</th>
<th>(q_B)</th>
<th>(e_B)</th>
<th>(t_P)</th>
<th>(q_P)</th>
<th>(e_P)</th>
<th>(y_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2403</td>
<td>3.1623</td>
<td>0.3333</td>
<td>1.6992</td>
<td>2.1937</td>
<td>0.3333</td>
<td>1.6992</td>
<td>-0.6937</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0.4404</td>
<td>4.9632</td>
<td>0.2888</td>
<td>1.9648</td>
<td>2.1605</td>
<td>0.3333</td>
<td>1.5023</td>
<td>0.9938</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.5873</td>
<td>6.2857</td>
<td>0.2515</td>
<td>2.2274</td>
<td>2.0476</td>
<td>0.3333</td>
<td>1.4379</td>
<td>1.2143</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.7021</td>
<td>7.3191</td>
<td>0.2178</td>
<td>2.5166</td>
<td>1.8865</td>
<td>0.3333</td>
<td>1.4052</td>
<td>1.3865</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.7939</td>
<td>8.1448</td>
<td>0.1864</td>
<td>2.8640</td>
<td>1.6908</td>
<td>0.3333</td>
<td>1.3856</td>
<td>1.5241</td>
<td>7.4869</td>
</tr>
<tr>
<td>5</td>
<td>0.8668</td>
<td>8.8008</td>
<td>0.1560</td>
<td>3.3179</td>
<td>1.4668</td>
<td>0.3333</td>
<td>1.3727</td>
<td>1.6335</td>
<td>7.5894</td>
</tr>
<tr>
<td>6</td>
<td>0.9228</td>
<td>9.3050</td>
<td>0.1261</td>
<td>3.9707</td>
<td>1.2175</td>
<td>0.3333</td>
<td>1.3642</td>
<td>1.7175</td>
<td>7.8556</td>
</tr>
<tr>
<td>7</td>
<td>0.9629</td>
<td>9.6654</td>
<td>0.0960</td>
<td>5.0397</td>
<td>0.9442</td>
<td>0.3333</td>
<td>1.3587</td>
<td>1.7775</td>
<td>8.3165</td>
</tr>
</tbody>
</table>

Following Proposition 2.5, there is income stratification with the rich living in Barrington and the poor living in Providence. The indifferent agent is \(i_b\) and \(y_b\) is his income. We observe that as \(c\) increases, Barrington becomes a more exclusive community and the indifferent agent becomes richer. The tax rate in Providence, \(t_P\), is higher than the tax rate in Barrington, \(t_B\) (Proposition 2.6). Note that as \(c\) increases, the tax rate in Barrington decreases while the tax rate in Providence does not change. Consistent with the same proposition, the quality of education in Barrington, \(q_B\), is higher than the quality of education in Providence, \(q_P\), except when \(c = 0\), in which case both cities provide the same quality. We also observe that as \(c\) increases, the quality of education improves considerably in Barrington, while it deteriorates in Providence. Expenditure per student, \(e_B\), starts higher in Barrington but decreases as \(c\) increases and sinks below the corresponding expenditure per student in Providence, \(e_P\). These results contrast with part of the previous literature, where income stratification leads to a combination of higher tax rates and higher quality of education in richer communities (e.g., Fernandez and Rogerson (1996), Westhoff (1977)) and agrees with Fernandez and Rogerson (1997). For the moment, ignore the column labeled \(y_0\). \(^5\)We use Mathematica for our computations. The programs used to produce the results in the paper are available from the authors upon request.
2.3.2 Example 2

Assume now that preferences can be represented by a quasilinear utility function, \( u(x, q) = x + \ln(q) \). As in the previous example, the production function is given by: \( g(z) = z^\alpha, \quad 0 < \alpha < 1 \). Moreover, the distribution of the initial endowment of the private good \( y \) is continuous with positive support: \( y \in [y_l, y_h] \subseteq \mathbb{R}_{++} \).

**Proposition 2.7**

1. Preferences on \( t \) are single peaked.
2. In every equilibrium community, the median voter is the agent with median income.
3. (a) If \( t_B < t_P \), then the rich live in Barrington and the poor live in Providence.
   (b) If \( t_B > t_P \), then the rich live in Providence and the poor live in Barrington.
4. In every equilibrium there is an individual who is indifferent between the two communities.

*Proof:* The proof is analogous to the proof of Proposition 2.5 and it is left to the reader. \( \square \)

The following Proposition shows under which conditions the high quality – low tax pattern arises in equilibrium. More specifically, when the poor live in Providence and the rich live in Barrington, whether or not Barrington provides better education depends on the difference between the fixed cost, \( c \), of living in Barrington and the amount that the indifferent individual saves by living in Barrington due to the lower tax rate.

**Proposition 2.8** Let \( (P, t_P, q_P), (B, t_B, q_B) \) be a stratified equilibrium and let \( y_b \) be the income of an individual who is indifferent between the two communities.

1. Assume \( t_B < t_P \). Then \( q_B > q_P \) iff \( c > (t_P - t_B)y_b \).
2. If \( t_B > t_P \) then \( q_B > q_P \).

*Proof:*

Consider the community of Providence \( (P, t_P, q_P) \). By Proposition 2.7, \( t_P \) must be the preferred tax rate of the agent with median income in Providence.

Solving for the median voter’s preferred tax rate, we get:

\[
t_P = \frac{\alpha}{y_{mP}^P},
\]

where \( y_{mP}^P \) is the income of the median agent in Providence.

Similarly, since \( (B, t_B, q_B) \) is a majority voting equilibrium, \( t_B \) must be the preferred tax rate of the agent with median income in Barrington:

\[
t_B = \frac{\alpha}{y_{mB}^B}.
\]

1. From Proposition 2.7, when \( t_P > t_B \) there is income stratification with the rich living in Barrington and the poor living in Providence.
By the same proposition, there is someone who is indifferent between living in Providence and living in Barrington. By definition of $y_b$,

\[(1 - t_P)y_b + \ln(q_P) = (1 - t_B)y_b - c + \ln(q_B)\]

\[\iff c - (t_P - t_B)y_b = \ln\left(\frac{q_B}{q_P}\right).\]

Now, $q_B < q_P$ if and only if $\ln(q_B/q_P) < 0$, if and only if $c - (t_P - t_B)y_b < 0$.

Therefore if $c > (t_P - t_B)y_b \Rightarrow q_B > q_P$.

2. From Proposition 2.7, when $t_P < t_B$ there is income stratification with the poor living in Barrington and the rich living in Providence. Again, by definition of $y_b$,

\[(1 - t_P)y_b + \ln(q_P) = (1 - t_B)y_b - c + \ln(q_B)\]

\[\iff (t_P - t_B)y_b = c + \ln\left(\frac{q_P}{q_B}\right).\]

Since $y_b \geq 0, c > 0, t_P < t_B$ then $q_P < q_B$. \hfill \square

We can now calculate the equilibrium when $i \in [0, 1]$ and $y(i) = 9i + 1$ or $y \sim U[1, 10]$ and $\alpha = 1/2$. We focus on equilibria with the rich living in Barrington and the poor living in Providence. The results are summarized in Table 2.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$i_b$</th>
<th>$y_b$</th>
<th>$t_B$</th>
<th>$q_B$</th>
<th>$c_B$</th>
<th>$t_P$</th>
<th>$q_P$</th>
<th>$c_P$</th>
<th>$y_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.2615</td>
<td>3.3532</td>
<td>0.0748</td>
<td>0.8228</td>
<td>0.5000</td>
<td>0.2297</td>
<td>1.3828</td>
<td>0.5000</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5048</td>
<td>5.5425</td>
<td>0.0643</td>
<td>1.0047</td>
<td>0.5000</td>
<td>0.1528</td>
<td>0.9953</td>
<td>0.5000</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>0.7507</td>
<td>7.7557</td>
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<td>1.4160</td>
<td>0.5000</td>
<td>0.1142</td>
<td>0.8161</td>
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<td>7.6000</td>
</tr>
<tr>
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<td>0.0524</td>
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<td>0.0994</td>
<td>0.7471</td>
<td>0.5000</td>
<td>8.1000</td>
</tr>
<tr>
<td>2.0</td>
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<td>9.6371</td>
<td>0.0509</td>
<td>3.5211</td>
<td>0.5000</td>
<td>0.0940</td>
<td>0.7218</td>
<td>0.5000</td>
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</tr>
<tr>
<td>2.5</td>
<td>0.9849</td>
<td>9.8640</td>
<td>0.0503</td>
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<tr>
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<td>0.5000</td>
<td>0.0913</td>
<td>0.7090</td>
<td>0.5000</td>
<td>9.7403</td>
</tr>
</tbody>
</table>

As before, there is income stratification. The indifferent agent is $i_b$ and his endowment is $y_b$. As in the previous example, Barrington becomes a more exclusive community as $c$ increases. The tax rate in Providence, $t_P$, is higher than the tax rate in Barrington, $t_B$. As $c$ increases, the tax rate in both communities decreases, since the median voter is now richer in both communities and with quasilinear preferences richer individuals prefer lower taxes. As predicted by Proposition 2.8, the quality of education in Barrington, $q_B$, is higher than the quality of education in Providence, $q_P$, except when $c$ is very small,
in which case the quality of education in Providence is higher. As \( c \) increases, the quality of education improves in Barrington and worsens in Providence. In this particular example, expenditure per student is the same in both communities and remains constant with \( c \).

3 Vouchers

3.1 The inefficiency of an economy without vouchers

One of the features of the model described in the previous section is that individuals are compelled to consume the education provided by the community in which they live. Such a restriction is likely to lead to an inefficient outcome, an inefficiency that is one of the reasons for the proposed voucher schemes. A lucid and common criticism of the state of affairs described by the equilibrium in the above model can be found in Friedman (1962), who observed that individuals who live in poor communities get a lower quality of education than those who live in richer communities although they are willing to pay the full cost of education in the latter community. In other words, a poor resident of a poor community may be willing to pay for tuition in better schools but may not be willing to move to the richer community. Children of poor parents may be trapped in low quality schools because it is too costly for their parents to relocate to the other community, and not because they are unwilling to buy better education. To demonstrate such a scenario we shall build a voucher system combined with a price system that leads to a Pareto improvement with respect to the equilibrium defined in the previous section.

Consider an equilibrium \((P, t_P, q_P), (B, t_B, q_B)\). We want to introduce a voucher program in Providence enabling its residents to choose between sending their children to the local public school or getting the per capita expenditure on education and using it to buy education in Barrington’s public school system. Formally, let \( e_P = E_P / \lambda(P) \) be the per capita expenditure on public education in Providence. The level of the voucher will be

\[
v = e_P.
\]  

(3.1)

Since we want to show that a Pareto improvement is feasible, we want the quality of education in Barrington not to deteriorate as a result of the flow of newcomers from the neighboring city. To this end, non-Barringtonians who want to use Barrington public schools will be charged a price that allows for the necessary expansion in Barrington’s expenditure on education. More formally, letting \( P_v \) be the set of Providence inhabitants who choose to use the voucher, the price \( p \) charged by Barrington is implicitly defined by the following equation:

\[
q_B = \frac{g[E_B + p\lambda(P_v)]}{\lambda(B) + \lambda(P_v)}.
\]  

(3.2)

This price is chosen to provide the resources necessary to keep the quality of education in Barrington unchanged. Naturally, any difference between the price \( p \) and the voucher \( v \) must be financed by the individuals who live in Providence and choose to send their children to study in Barrington, out of their own pockets.

The introduction of vouchers brings about some changes in the Providence budget that need to be taken into account. On the one hand, given the tax rate \( t_P \), the total resources available for education are reduced by the amount \( v\lambda(P) \). On the other hand, less people consume Providence schooling. As a
result, the new quality of education in Providence is given by

\[ q_P = \frac{g[E_P - v\lambda(P_v)]}{\lambda(P \setminus P_v)}. \tag{3.3} \]

Since the expenditure per capita remains the same and \( g \) is a concave function with \( g(0) = 0 \), the quality of education in Providence does not deteriorate: \( q_P \geq q_P^6 \).

Given the equilibrium \( ((t_P, q_P), (B, t_B, q_B)) \) without vouchers, there is a Pareto improvement if we can find a tuition fee \( p \) and a subset \( P_v \subset P \) of positive measure such that \( v \) and \( p \) satisfy equations (3.1), (3.2) and (3.3), respectively, and for all \( i \in P_v, \)

\[ u[(1 - t_P)y(i), q_P] \leq u[(1 - t_P)y(i) + v - p, q_B]. \tag{3.4} \]

Consider the case of income stratification, with the poor living in Providence and the rich in Barrington:

\[ P = y^{-1}([y_l, y_b]) \quad \text{and} \quad B = y^{-1}((y_b, y_h)). \]

Further assume that only the richest inhabitants of Providence use the voucher: \( P_v = y^{-1}([y_0, y_b]) \). In this case, equations (3.2) and (3.3) become, respectively:

\[ q_B = \frac{g\left(E_B + p \int_{y_0}^{y_b} dF\right)}{\int_{y_0}^{y_b} dF} \tag{3.2'}, \]

\[ q_P = \frac{g\left(e_p \int_{y_0}^{y_b} dF\right)}{\int_{y_0}^{y_b} dF} \tag{3.3'}. \]

The endowment \( y_0 \) of an individual who is indifferent between the voucher and Providence public school satisfies

\[ u[(1 - t_P)y(i), q_P] = u[(1 - t_P)y(i) + v - p, q_B]. \tag{3.4'} \]

Look at Figure 1. It depicts some examples of a Pareto improvement. The downward sloping curve, \( \text{Price} \), represents the function implicitly defined by equation (3.2'), namely, the price \( p \) that Barrington residents charge individuals in \( P_v \) as a function of \( y_0 \), so that their education quality remains unchanged. Fewer people using the vouchers (higher \( y_0 \)) implies that newcomers will be charged a lower price.

The upward sloping curve, \( \text{Indif} \), represents the price \( p \) of the alternative education that makes a person with endowment \( y_0 \) indifferent between using the voucher and using the Providence public system. This is implicitly defined by equation (3.4'), after substituting it into equation (3.3').

If these two curves intersect, the intersection point \( (y_0, p) \) corresponds to a Pareto improvement with respect to the original equilibrium without vouchers: (i) if individuals with endowment in \([y_0, y_b]\) opt for using the vouchers, \( p \) is the price necessary to keep the quality of education in Barrington constant; and (ii) if \( p \) is the price charged by Barrington residents, then individuals with endowment in \([y_0, y_b]\) are those who are willing to opt for the vouchers.

\[ ^6 \text{Note that } q_P = \frac{g(E_P)}{E_P}v \text{ and } q_P' = \frac{g(E_P - v\lambda(P_v))}{E_P - v\lambda(P_v)}v. \]
Reverting to Tables 1 and 2, it can be seen that when Barrington is exclusive enough, vouchers are used by a certain percentage of the other city’s population. The last column, \( y_0 \), represents the endowment of a Providence resident who is indifferent between using the voucher and attending a Providence public school. For the example with the logarithmic utility function, the voucher is used when \( c > 3 \); for the quasilinear utility function, this happens when \( c \geq 1 \). As this example suggests, the equilibria without vouchers are inefficient.

![Figure 1: Vouchers used by the rich in Providence.](image)

3.2 Equilibrium with vouchers

In the former section we showed that the equilibrium without vouchers is inefficient. However, our examples were not an equilibrium. If a voucher scheme is introduced in Providence, some residents of Barrington might want to move to Providence, so as to save the fixed cost \( c \) of living in Barrington while enjoying the same education quality.

In this section we look for an equilibrium with vouchers that Pareto dominates the equilibrium without vouchers.

We must first define a community with a voucher system. This will be a group of agents —some of whom will opt to use the voucher while the rest use the local public school— together with a tax rate, a voucher and a quality of education such that (i) both the voucher and the quality of education are financed by proportional income taxation and (ii) the voucher equals the expenditure on education per student. Formally:

**Definition 3.1** A community with a voucher system \((J, J_v, t_J, v_J, q_J)\) consists of:

- A measurable set \( J \subset I \) of positive measure of individuals who live in the community.
- A measurable subset \( J_v \subset J \), of individuals who use the voucher, such that \( \lambda(J \setminus J_v) > 0 \).
- An income tax rate \( t_J \in [0, 1] \).
- A voucher \( v \) such that:
  \[
  v = v(t_J) \equiv \frac{t_J \int_J y(i) d\lambda}{\lambda(J)} , \tag{3.5}
  \]
  i.e., the voucher equals per capita expenditure on education.
A quality of education \( q_J \) such that:

\[
q_J = q(t_J, J, J_v) \equiv \frac{g[t_J \int_J y(i) d\lambda - v\lambda(J_v)]}{\lambda(J \setminus J_v)},
\]

\[ (3.6) \]

i.e., \( q_J \) satisfies the budget constraint.

Next, as in the case of no vouchers, we require that the tax rate be decided by majority vote.

**Definition 3.2** Let \((J, J_v, t_J, v, q_J)\) be a community with a voucher system. Let \( q \) be the quality of an alternative education system and let \( p \) be its price. For each \( t \in [0, 1] \), let \( J_v(t) \) be the set of agents who use the vouchers and prefer the tax rate \( t \) to \( t_J \):

\[
J_v(t) = \{ i \in J_v : u[(1-t)y(i) - c_J + v(t) - p, q] > u[(1-t_J)y(i) - c_J + v - p, q] \}.
\]

Similarly, for each \( t \in [0, 1] \), let \( J_{nv}(t) \) be the set of agents who do not use vouchers and prefer the tax rate \( t \) to \( t_J \):

\[
J_{nv}(t) = \{ i \in J \setminus J_v : u[(1-t)y(i) - c_J, q(t_J, J, J_v)] > u[(1-t_J)y(i) - c_J, q_J] \}.
\]

We say that \((J, J_v, t_J, v, q_J)\) is a majority voting equilibrium with respect to \( p \) and \( q \) if

\[
\lambda(J_v(t) \cup J_{nv}(t)) \leq \frac{1}{2} \lambda(J), \quad \forall t \in [0, 1].
\]

We can now define the equilibrium concept that we will use.

**Definition 3.3** Let \( E = (u, y, g, c) \) be an economy. A voucher equilibrium of \( E \) is a partition of \( I \) into a community with a voucher system \((P, P_v, t_P, v, q_P)\) and a viable community \((B, t_B, q_B)\), and a price \( p \) that satisfy:

1. **Majority voting.** \((P, P_v, t_P, v, q_P)\) is a majority voting equilibrium with respect to \( p \) and \( q_P \). \((B, t_B, q_B)\) is a majority voting equilibrium in Barrington.

2. **Non-deterioration of education in Barrington:**

\[
q_B = \frac{g[E_B + p\lambda(P_v)]}{\lambda(B) + \lambda(P_v)}.
\]

3. **Free mobility.** Every agent is happy with his choice of community and education.

(i) \( \forall i \in P \setminus P_v, \)

\[
u[(1-t_P)y(i), q_P] \geq \max\{u[(1-t_P)y(i) + v - p, q_B], u[(1-t_B)y(i) - c, q_B]\}.
\]

(ii) \( \forall i \in P_v, \)

\[
u[(1-t_P)y(i) + v - p, q_B] \geq \max\{u[(1-t_P)y(i), q_P], u[(1-t_B)y(i) - c, q_B]\}.
\]
(iii) \( \forall i \in B, \)
\[
u[(1 - t_B)y(i) - c, q_B] \geq \max\{u[(1 - t_P)y(i), q_P], u[(1 - t_P)y(i) + v - p, q_B]\}.
\]

3.3 Examples

In this section we consider the particular functional forms of the examples introduced above and calculate the equilibrium of these particular economies in the new framework with vouchers.

3.3.1 Example 1 (cont.)

Recall that the common preferences are represented by the utility function \( u(x, q) = \ln(x) + \ln(q) \) and that production of education exhibits decreasing returns: \( g(z) = z^\alpha, \ 0 < \alpha < 1 \). Moreover, the distribution of the initial endowment of the private good is continuous with positive support: \( y \in [y_l, y_h] \subseteq \mathbb{R}_{++} \).

We now introduce a proposition that will help us characterize and calculate equilibrium allocations: in an equilibrium where the voucher is no higher than the price charged by the absorbing community, individuals can be partitioned into three consecutive income brackets: the poorest live in Providence and use its public schools, the middle level lives in Providence and uses vouchers and the richest live in Barrington.

**Proposition 3.1** Let \( ((P, P_v, t_P, v, q_P), (B, t_B, q_B), p) \) be an equilibrium with vouchers where \( p > v \). Then there is income stratification in the following sense:

1. \( y(i) < y(j) \), for all \( i \in P \setminus P_v \) and \( j \in P_v \),
2. \( y(k) < y(l) \), for all \( k \in P \) and \( l \in B \).

**Proof:**

1. Let
\[
w(y) = \ln[(1 - t_P)y + v] + \ln(q_P) - \ln[(1 - t_P)y + v - p] - \ln(q_B),
\]
\[w(y) = \ln[(1 - t_P)y + v] + \ln(q_P) - \ln[(1 - t_P)y + v - p] - \ln(q_B),
\]
where \( y \in [y_l, y_h] \).

2. From the previous item we know that the richest individuals in Providence use the voucher. Let
\[
v(y) = \ln[(1 - t_p)y + v - p] + \ln(q_B) - \ln[(1 - t_B)y - c] - \ln(q_B)
\]
be the utility differential between living in Providence using the voucher and living in Barrington. The result follows after noting that
\[
\frac{dv(y)}{dy} = \frac{1 - t_P}{(1 - t_P)y + v - p} - \frac{1 - t_B}{(1 - t_B)y - c}
\]
We denote the borders of the income brackets determined in the previous proposition by \( y_l, y_v, y_b, \) and \( y_h \). More graphically, an equilibrium partition should look as follows:

\[
\begin{array}{c}
\hline
P & B \\
\hline
y_l & y_v & y_b & y_h \\
\hline
\end{array}
\]

**Figure 2.**

We now compute equilibria with vouchers and compare them with the corresponding equilibria without vouchers. Since the range of the income function \( y \) is convex, it can be shown (as in Proposition 2.2) that there exists an individual with endowment \( y_b \), who is indifferent between living in Providence and living in Barrington:

\[
u[(1 - t_P)y_b + v - p; q_B] = u[(1 - t_B)y_b - c; q_B]. \tag{3.8}
\]

By Proposition 3.1 we know that the richest individuals in Providence use vouchers, and therefore there is a resident of Providence (whose income is \( y_v \)) who is indifferent between using the local public school or the voucher:

\[
u[(1 - t_P)y_v + v - p, q_B] = u[(1 - t_P)y_v, q_P]. \tag{3.9}
\]

We now turn to the determination of the communities’ tax rates. Consider the community of Providence, \( (P, P_v, t_P, v, q_P) \) and an individual \( i \in P \setminus P_v \) with initial endowment \( y \). His preferences over taxes are represented by the following function:

\[
\ln[(1 - t)y] + \ln \left[ \frac{g \left( \frac{\lambda(P \setminus P_v)}{\lambda(P)} \int \mu_P y(i) d\lambda \right)}{\lambda(P \setminus P_v)} \right].
\]

These preferences are single peaked. The agent’s preferred tax rate is:

\[t^*(y) = \frac{\alpha}{1 + \alpha}.
\]

Consider now an agent \( i \in P_v \) with endowment \( y \). Let \( \mu_P \) be the mean income in community \( P \). Note that the value of the voucher is simply \( v_P = t_P \mu_P \). Individual \( i \)’s preferences over taxes are represented by:

\[\ln [(1 - t)y + t\mu_P - p] + \ln(q_B),\]
which are single peaked. His preferred tax rate is:

\[ t^*(y) = \begin{cases} 1 & \text{if } y < \mu_P, \\ 0 & \text{if } y > \mu_P. \end{cases} \]

Since preferences are single peaked, taxes are decided by the median voter. However, the median voter is no longer the agent with median income in the community. From Proposition 3.1, we know that the relatively rich residents of community \( P \) use vouchers. If the agents who use vouchers constitute less than half of the population in the community — an assumption that will be satisfied in our examples — then the preferred tax rate of the community’s median voter is

\[ t_P = \frac{\alpha}{1 + \alpha}. \tag{3.10} \]

Note that assuming that the Providence median voter does not use the voucher, means that the resulting tax rate is independent of the median voter’s income. This allows us to calculate the equilibrium without calculating the median voter’s income.

Since Barrington does not issue vouchers, the determination of the equilibrium tax rate is as in Section 2.3. Median income in Barrington, \( y^m_B \), is determined by

\[ \frac{\int_{y_B}^{y^m_B} y \, dF}{\int_{y_B}^{y^h} y \, dF} = \frac{1}{2}, \tag{3.11} \]

and the corresponding tax rate is

\[ t_B = \frac{\alpha}{1 + \alpha} \left( 1 - \frac{c}{y^m_B} \right). \tag{3.12} \]

By construction, the voucher in Providence is given by its per capita tax revenue:

\[ v = \frac{t_P \int_{y_P}^{y^m_P} y \, dF}{\int_{y_P}^{y^h} y \, dF}. \tag{3.13} \]

On the other hand, the price \( p \) is such that new users do not affect the quality of education of Barrington:

\[ g \left( t_B \int_{y_B}^{y^m_B} y \, dF + p \int_{y_e}^{y^m_e} y \, dF \right) \int_{y_B}^{y^m_B} y \, dF = q_B. \tag{3.14} \]

Given the budget constraint in each community, education quality can be calculated as:

\[ q_P = g \left( t_P \int_{y_P}^{y^m_P} y \, dF - v \int_{y_P}^{y^m_P} y \, dF \right) \int_{y_P}^{y^m_P} y \, dF, \]

\[ q_B = g \left( t_B \int_{y_B}^{y^m_B} y \, dF \right) \int_{y_B}^{y^m_B} y \, dF. \tag{3.15} \]

In order to find an equilibrium, we simply have to solve the former system of nine equations, (3.8)–(3.15), with nine unknowns: \( (y_B, y_e, t_P, y^m_B, t_B, v, p, q_P, q_B) \).
Table 3 and Figure 3 summarize the equilibrium of the numerical example where $\alpha = 1/2$, $c = 4$ and $y \sim U[1,10]$.

**Table 3: Comparing Equilibrium With and Without Vouchers**

<table>
<thead>
<tr>
<th></th>
<th>$y_b$</th>
<th>$y_v$</th>
<th>$t_B$</th>
<th>$q_B$</th>
<th>$p$</th>
<th>$t_P$</th>
<th>$q_P$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No voucher</td>
<td>8.1448</td>
<td>-</td>
<td>0.1863</td>
<td>2.8640</td>
<td>-</td>
<td>0.3333</td>
<td>1.3855</td>
<td>-</td>
</tr>
<tr>
<td>Voucher</td>
<td>8.4536</td>
<td>7.6819</td>
<td>0.1888</td>
<td>3.1843</td>
<td>4.3554</td>
<td>0.3333</td>
<td>1.4567</td>
<td>1.5756</td>
</tr>
</tbody>
</table>

As anticipated, slightly more people now live in Providence compared with the equilibrium without vouchers (3.5% more). A non-trivial proportion of those living in Providence use vouchers: 8.57% of the total population and 10.35% of those who live in Providence. The quality of education increases in both communities. In Providence, the reduction in the number of users of the public school system dominates the loss of resources, owing to decreasing returns in the production of education. In Barrington, education quality increases because the community has become slightly more exclusive and because, by construction, voucher users do not directly affect education quality. Note, however, that the difference in the quality of education provided by the two communities increases.

Figure 4 depicts the utility differential between the equilibrium with vouchers and the equilibrium without vouchers for all agents in the economy. Utility increases for everybody and therefore the equilibrium with vouchers Pareto dominates the equilibrium without vouchers. The figure is self-explanatory.
We would like to emphasize that the main results in the former example would obtain in a model with no crowding for an appropriate value of the fixed cost, $c$. The main difference is that if $c$ is either too low or too high in the benchmark model, one of the two communities may be empty. It could be shown in a completely analogous way that the proposed voucher system would lead to a Pareto improvement as an equilibrium phenomenon for an appropriate $c$, which produces non-empty communities. On the other hand, when there are no crowding effects, the difference in quality across communities does not increase when introducing vouchers; it stays constant.

### 3.3.2 Example 2 (cont.)

Now assume that preferences can be represented by the quasilinear utility function $u(x, q) = x + \ln(q)$ and that production of education is as before: $g(z) = z^\alpha$, $0 < \alpha < 1$, i.e. $g$ exhibits decreasing returns. The income function $y$ that represents the initial endowment of the private good is continuous with positive support.

Consider a community with a voucher system, $(P, P_v, t_P, v_p)$, and let $q$ be the quality of an alternative education which can be purchased at price $p$. Assume further that the income distribution in community $P$ is such that the alternative education is affordable by each and every agent in $P$. Since preferences are quasilinear, it is easy to see that if one agent prefers to use the voucher, then all agents will prefer to do so. Therefore, an equilibrium with vouchers will not be characterized by only rich individuals in the community using the voucher. In fact, any subset of inhabitants that has the appropriate measure can, in equilibrium, use the vouchers. In particular, quasilinear preferences allow us to implement a voucher system aimed at the poor, i.e., $P_v = y^{-1}([y_l, y_u])$.

Adapting the equations of the previous example to the case of our quasilinear utility function, with $\alpha = 1/2$, $c = 1.4$ and $y \sim U[1, 10]$, we get the results summarized in Table 4 and Figure 5. The only practical difference from the previous example is that in this case, since the preferred tax rate of individuals who do not use the vouchers depends on their income, we cannot avoid calculating the income of the Providence median voter. Here, as long as the equilibrium proportion of the population that uses the
voucher does not exceed one half, the income of the median voter is the median income in the community.\footnote{Agents $i \in P_v$ have a preferred tax rate $t^* = 1$; agents $j \in P \setminus P_v$ have $t^* = \alpha/y$. If $\lambda(P_v) \leq 1/2$, then $t_P = \alpha/y_P$.}

### Table 4: Equilibrium with and without vouchers

**Quasilinear Utility $c = 1.4$**

<table>
<thead>
<tr>
<th></th>
<th>$y_b$</th>
<th>$y_v$</th>
<th>$t_B$</th>
<th>$q_B$</th>
<th>$p$</th>
<th>$t_P$</th>
<th>$q_P$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No vouchers</td>
<td>8.719</td>
<td>-</td>
<td>0.0529</td>
<td>1.9972</td>
<td>-</td>
<td>0.1012</td>
<td>0.7560</td>
<td>-</td>
</tr>
<tr>
<td>Vouchers for poor</td>
<td>8.9951</td>
<td>1.9518</td>
<td>0.0526</td>
<td>2.1161</td>
<td>1.4735</td>
<td>0.1000</td>
<td>0.7993</td>
<td>0.5000</td>
</tr>
</tbody>
</table>

We see that more people live in Providence when the voucher system is available than when it is not. A non-trivial proportion of people uses vouchers: 11.9\% of those living in Providence (or 10.5\% of the total population). Taxes slightly decrease in both communities since both median voters are slightly richer. The quality of education increases in both communities but the quality difference still increases. Figure 6 shows that utility increases for everybody in the economy. The equilibrium with vouchers Pareto dominates the equilibrium without vouchers.

\footnote{Agents $i \in P_v$ have a preferred tax rate $t^* = 1$; agents $j \in P \setminus P_v$ have $t^* = \alpha/y$. If $\lambda(P_v) \leq 1/2$, then $t_P = \alpha/y_P$.}
Note that in this case, since expenditure per capita is the same across communities, a voucher system in the model with no crowding would not produce any benefit.

4 Summary and Conclusions

We develop a simple two-good, two-community model that helps us study some of the effects that a voucher system may have on the quality of public education. We show that the introduction of a voucher system may result in a Pareto improvement as an equilibrium outcome. That is, we show not only that the equilibrium of the economy without vouchers is inefficient, but also that the introduction of vouchers in itself, and without the aid of additional money transfers, brings about a state of affairs that is preferred by all members of society.

Needless to say, the model is very simple and could be improved in several directions. First, we have focused on public education, ignoring private alternatives. Several models consider the consequences of a private sector for education. Without trying to be exhaustive, see for example Stiglitz (1974), Ireland (1990), Glomm and Ravikumar (1998) or Epple and Romano (1996). Secondly, we have agents who differ only in income, therefore we abstracted from peer group effects. In our story, education quality relates to resources devoted to education and number of students only. We do not consider the composition of the student body. Several authors discuss the importance of peer group effects in education (for example Henderson, Mieszkowski, and Sauvageau (1978), de Bartolomé (1990), Epple and Romano (1998) or Manski (1992)). Third, we do not consider housing markets. As explained in the introduction, property taxes are the main source of revenue to pay for local education. However, we choose to use income taxes to keep the model tractable (see Epple and Romer (1991)). Finally, the model that we present is static in nature. Recently Fernandez and Rogerson (1998) have analyzed the dynamic implications of education reforms.

As mentioned before, models abstract from some relevant issues and focus on some others. The purpose of the paper is simply to show that a voucher system that applies only to public schools can result in a Pareto improvement. This Pareto improvement would lead to higher education quality in both communities but also to higher education quality differentials. We hope this result can help in
understanding how a voucher system can affect educational outcomes.
References


