1. Introduction

U.S. LFP of women has been lower than that of men, and gradually increasing since 1890. Since Mincer’s 1962 seminal work, the explanation for these phenomena focuses on the observed gradual increase in women wages. Along this line, Galor and Weil (1996) construct a general equilibrium model in which the increase in women’s wages and LFP is a by-product of the process of development in which capital accumulation raises women’s wages and therefore female LFP. The accumulation of capital also reduces the relative returns to physical strength and therefore narrows the gender gap. Greenwood, Seshardi and Yorukoglu (2001) focus too on the rise in overall real wages, in conjunction with the introduction of labor-saving household appliances, as the engine behind the rise in female LFP.

Seemingly contradicting with this approach is the empirical finding regarding the dynamic pattern of the ratio of the mean wage of working women to the mean wage of working men, often referred to as the “gender gap in wages”. Data taken from Smith and Ward (1989) and O’Neil and Polachek (1993) show that this ratio has presented a non-monotonic dynamics during the 20th century.

So far, there is no theoretical paper offering an explanation for these non-monotonic dynamics of the gender gap. Our purpose in this paper is to fill this void. In our model, the wage per unit of labor is continuously increasing due to capital accumulation. This entails a continuing growth in women’s LFP, as in Galor and Weil (1996). The non-monotonic dynamics of the means ratio springs from the evolution of the compositions of the populations of working men and women.

Our explanation is related to the empirical works of Smith and Ward (1989), Goldin (1990) and O’Neil and Polachek (1993). These studies distinguish between two kinds of periods. In periods of the first type, a sufficiently large number of new entrants

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1 See the data in the Handbook of Labor Economics (1986).
were married women older than about 35 years with low skills and little labor market
experience. The new entrants have lowered the average market skills of working
women, slowing down the growth in average women earnings, and making it grow
slower than men’s. In periods of the second type the dominant factor in the expansion in
female LFP was the rise in the participation rates of younger women. In those periods,
due to the early participation, women’s experience has approached that of men and the
main reason for the decrease in the ratio of mean wages was removed.

The main contribution of our paper is the analysis of the individuals’
considerations underlying their observed labor choices. In particular we conclude that a
massive entry of women in a relatively late stage of their lives into the labor force
occurs in periods in which wages grow sufficiently rapidly. In such periods the wage a
woman faces when she is young is much lower than the wage when she is older. This
difference in wages may lead to differences in her LFP choices over time. In periods in
which wages grow sufficiently slow, the similarity in wages during the different periods
of the woman’s life leads to similarity in her LFP decisions in those periods.

We present an overlapping generations model in which individuals live and
work for two periods. The economy comprises three production sectors: home
production, a physical sector and a modern sector. At home and in the physical sector
labor is the sole input, while the modern sector production utilizes both labor and
capital. We refer to employment in the physical and the modern sectors as labor market
participation. We assume that individuals differ in their ability and that the individual’s
ability affects the amount of efficiency units of labor that she or he can supply to the
modern sector. The distribution of ability among women is identical to that among men.
An important assumption in the model is that by entering the labor force early in life,
individuals acquire experience that increases their amount of efficiency units later in
life. Several simplifying assumptions regarding productivity at home and in the physical
sector ensure that women never choose to work in the physical sector and men never
choose home production. We further assume that men working in the physical sector
make more than women working at home. Since women’s alternative to working in the modern sector is lesser than men’s, the average ability and the average income of women in the modern sector are lower than those of men.

Due to these assumptions, in the initial stage of the economy’s growth, women labor follows three alternative dynamic labor profiles: The least able women work at home in both their lives’ periods; abler women work at home in the first period of their life and in the modern sector in the second period; the ablest women work in the modern sector in both periods. The middle group exists because in this initial stage wages grow rapidly attracting men and women in their second period of life to the modern sector. Since these men have labor market experience while these women do not, the mean income of men rises more than the mean income of women.

As the wages growth decelerates, the middle group disappears and with it the differences in experience between men and women of the same generation in the modern sector. The growth of wages in the modern sector affects women average income more than it affects men’s, since female LFP consists only of work in the modern sector while men also work in the physical sector. Thus, in this stage the gender gap narrows.

The paper is organized as follows. In section 2 we present the basic structure of the model. In section 3 we analyze the individuals’ labor supply decisions. In section 4 we calculate the equilibrium and the dynamics of the economy portrayed by the model. In section 5 we compare the dynamics of the gender gap implied by the model with the empirical data. We show that if, following Goldin (1991), we interpret World War II as a temporary decrease in the value of home production then the model yields W-shaped gender gap dynamics for the US during the twentieth century. The data in Smith and Ward (1989), Goldin (1990) and O’Neil and Polachek (1993) indeed shows such W-shaped dynamics. In section 6 we provide concluding remarks.
2. The Structure of the Economy

Consider a closed overlapping generations economy that operates in a perfectly competitive environment. Time is discrete and infinite. In every period the economy produces a single good that can be used for consumption or investment.

2.1 Production

Production can take place at home or in the market. There are two production sectors in the market: the physical sector and the modern sector. Working in the market, in either sector, in life’s first period rewards the individual with experience that increases her or his productivity in the market production in life’s second period.

The marginal productivity of labor at home is the constant $H$, regardless of gender and experience. In contrast, the marginal productivity of labor at the physical sector differs across the genders. Each man in the physical sector produces the constant amount $P$ if he is inexperienced, or $\theta P$ if he is experienced, where $\theta$ is a constant greater than one. Likewise, women in the physical sector produce the constant amounts $P'$ or $\theta P'$, depending on experience.

The production function in the modern sector is:

\[
Q_t = K_t^{0.5} L_t^{0.5}
\]

Where $Q_t$ is output, $K_t$ is the amount of capital and $L_t$ is the amount of efficiency units of labor in this sector in period $t$.\(^2\)

Markets are assumed to be competitive. Hence the wage of one unit of efficiency labor in period $t$ and the return to one unit of capital in period $t$ are respectively:

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\(^2\) The specific value of 0.5 in the exponent of the production function is chosen to enable closed form solution for the variables of the model.
\[
(2) \quad w_t = 0.5 K_t^{0.5} L_t^{-0.5}
\]

\[
(3) \quad R_t = 0.5 K_t^{-0.5} L_t^{0.5}
\]

Thus:

\[
(4) \quad R_t = \frac{1}{4w_t}
\]

2.2 Individuals

In each period, a generation of measure 2 joins the economy where the measures of the women and the men in each generation are normalized to 1. All individuals live for two periods and work in both their life periods. For simplicity, we assume that individuals derive utility only from consumption in their second period of life. Due to this assumption, individuals maximize the net present value of their earnings.

Individuals differ in the amount of efficiency units. Let \(a_j\) be the amount of efficiency units that individual \(j\) has as an inexperienced worker. We assume \(a \sim U[0,1]\) regardless of gender. We further assume that as an experienced worker \(j\) can supply \(\theta a_j\) efficiency units to the modern sector.\(^3\)

We assume that \(P>H\). This assumption ensures that men do not work at home. We further assume that \(P'<H\) and also that \(\theta P'<H\). This ensures that women do not work in the physical sector.

In each period, four groups exist in the economy: women in their life’s first period, women in their life’s second period, men in their life’s first period, and men in

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\(^3\) For the results of the paper it is merely essential that working in the physical sector provides the worker with some additional productivity due to market experience. The extreme assumption that both working in the physical and the modern sectors have the same experience effect on future productivity in the modern sector is taken for simplicity sake.
their life’s second period. We assign the letters $A$, $B$, $C$ and $D$ to represent these groups respectively. We denote by $x_i^z$ the number of individuals that work in the modern sector in period $t$ who belong to group $z$, and by $L_i^z$ the amount of efficiency units of labor supplied to the modern sector by members of group $z$, where $z \in \{A, B, C, D\}$. From the uniform distribution of individuals efficiency units it follows that:

\[
L_i^z = x_i^z \left(2 - x_i^z\right) \over 2
\]

if all of the workers in group $z$ have no market experience.

### 3. Labor Supply

In this section we analyze the labor supply of each of the four groups of individuals that exists in the economy.

#### 3.1 Men’s Labor Supply

In each period $t$ man $j$ chooses to work in the modern sector if:

\[
a^jw_t \geq P
\]

Note that if (6) holds then $a^j\theta w_t \geq \theta P$, and thus, $j$ prefers the modern sector regardless of his experience. We can define therefore an ability threshold for men denoted by $a_i^Y$, where men having $a^j \geq a_i^Y$ work in the modern sector, and those having $a^j < a_i^Y$ work in the physical sector. The threshold $a_i^Y$ satisfies:
\[ a_t^\gamma = \begin{cases} \frac{P}{w_t}, & \text{if } \frac{P}{w_t} < 1 \\ 1, & \text{otherwise} \end{cases} \]  

(7)

Due to the uniform distribution of \( a \):

\[ x_t^c = x_t^d = 1 - a_t^\gamma = \begin{cases} 1 - \frac{P}{w_t}, & \text{if } \frac{P}{w_t} < 1 \\ 0, & \text{if } \frac{P}{w_t} \geq 1 \end{cases} \]  

(8)

The total amount of men in the modern sector is therefore \( x_t^c + x_t^d = 2x_t^c = 2x_t^d \). Given \( w_t \), the amount of efficiency units supplied in period \( t \) by men of generation \( t \) is:

\[ L_t^c = \int_{a_t^c} 1 - \left( a_t^\gamma \right)^2 = \begin{cases} \frac{1 - \left( \frac{P}{w_t} \right)^2}{2}, & \text{if } \frac{P}{w_t} < 1 \\ 0, & \text{otherwise} \end{cases} \]  

(9)

A man’s occupational choice in the second period of his life does not depend on his experience since experience increases his productivity by the same proportion, a multiplication by \( \theta \), in both relevant sectors. Therefore the occupational choice in period \( t \) of a man that was born in that period is isolated from his occupational choice in period \( t+1 \), and thus depends only on \( w_t \). Hence, given \( w_t \), the amount of efficiency units
supplied in period $t$ by men of generation $t-1$ is:

$$L^B_t = \theta \int_{a^*}^1 ada = \theta \left( \frac{1 - \left( \frac{P}{W_t} \right)^2}{2} \right) = \begin{cases} 0 & \text{if } \frac{P}{W_t} < 1 \\ \theta \frac{1 - \left( \frac{P}{W_t} \right)^2}{2} & \text{otherwise} \end{cases}$$

(10)

3.2 The Labor Supply of Women in Their Life’s Second Period

We define $a^X_t$ by $a^X_t \equiv 1 - x^A_t$. Thus, the women who do not acquire experience in period $t$ are the ones with $0 < a < a^X_t$, and the women who do are the ones with $a^X_t < a < 1$.

Three possible cases should be considered. In the first case, $a^X_t < \frac{H}{\theta W_{t+1}} < \frac{H}{W_{t+1}}$. In this case the women who did not acquire experience in period $t$, those with $0 < a < a^X_t$, do not work in the market in period $t+1$ since for them $a < \frac{H}{W_{t+1}}$, implying that $aw_{t+1} < H$.

Women who did acquire experience in period $t$ and their amount of efficiency units is in the range $a^X_t < a < \frac{H}{\theta W_{t+1}}$ also do not work in the market since for them $a \theta W_{t+1} < H$. The only women born in period $t$ who work in period $t+1$ are those with $\frac{H}{\theta W_{t+1}} < a < 1$. Thus,

$$x^B_{t+1} = 1 - \frac{H}{\theta W_{t+1}}$$

due to the uniform distribution of $a$. The labor supply of these women in this case is

$$L^B_{t+1} = \int_{\theta W_{t+1}}^1 ada = \frac{1 - \left( \frac{H}{\theta W_{t+1}} \right)^2}{2}.$$


In the second case, \( \frac{H}{\theta w_{t+1}} < a_i^X < \frac{H}{w_{t+1}} \). As in the previous case, the women who did not acquire experience in period \( t \), do not work in the market in period \( t+1 \) since for them \( a_0 \). On the other hand, all the women who did acquire experience in period \( t \) work in the market in period \( t+1 \), since for them \( a_a^X \) and therefore \( a > \frac{H}{\theta w_{t+1}} \). Thus, in this case \( x_{t+1}^B = x_{t+1}^A \). The period \( t+1 \) labor supply of women born in period \( t \) in this case

\[
L^B_{t+1} = \theta \int_{0}^{a} ada = \theta \int_{0}^{a} \frac{1}{2} - \frac{1 - a^X}{2} = \theta \int_{0}^{a} \frac{1}{2} - \frac{1 - a^X}{2} 
\]

In the third case, \( \frac{H}{\theta w_{t+1}} < \frac{H}{w_{t+1}} < a_i^X \). As in the second case, all the women who did acquire experience in period \( t \) work in the market in period \( t+1 \), for the same reason as in that case. In addition, some women born in period \( t \) who worked at home in period \( t \) will work in the market in period \( t+1 \) too. These women are the ones with \( \frac{H}{w_{t+1}} < a < a_i^X \). The rest of the women who were born in period \( t \) and worked at home in that period are the ones to whom \( 0 < a < \frac{H}{w_{t+1}} \) and therefore they work at home in period \( t+1 \) too. The period \( t+1 \) labor supply of women born in period \( t \) in this case

\[
L^B_{t+1} = \theta \int_{0}^{a} ada + \int_{a}^{a_i^X} \theta (\theta - 1) \frac{1}{2} - \frac{1 - a^X}{2} = \frac{H^2}{w_{t+1}} 
\]

3.3 The Labor Supply of Women in Their Life’s First Period

In contrast to men, the period \( t \) occupational choice of a woman born in period \( t \) depends not only on current wage, \( w_t \), but also on \( w_{t+1} \). The connection between the occupational choices in period \( t \) and period \( t+1 \) springs from the different effect that
experience has on productivity in modern sector and on productivity at home.

Each woman born in period $t$ has to choose in that period one of the following dynamic labor profiles:

Profile 1: Home production in period $t$ and in period $t+1$.
Profile 2: Home production in period $t$ and work in the modern sector in period $t+1$.
Profile 3: Work in the modern sector in period $t$ and home production in period $t+1$.
Profile 4: Work in the modern sector in period $t$ and in period $t+1$.

We define $V(i)$ as the present value of earnings under each of the profiles $i \in \{1, 2, 3, 4\}$.
Given $w_t$, $w_{t+1}$ and $R_{t+1}$, $V(i)$ satisfies:

$$V(1) = H + \frac{H}{R_{t+1}}$$
$$V(2) = H + \frac{aw_{t+1}}{R_{t+1}}$$
$$V(3) = aw_t + \frac{H}{R_{t+1}}$$
$$V(4) = aw_t + \frac{aR_{t+1}}{R_{t+1}}$$

Define $a_{mn,t}$ as the threshold of efficiency units of labor of an inexperienced female worker, born in period $t$, which for all $a > a_{mn,t}$ profile $m$ is preferred to profile $n$, where $m > n$ and $m, n \in \{1, 2, 3, 4\}$. The formulas for these thresholds as functions of $w_t$ and $w_{t+1}$ are presented in the appendix.

We start analyzing the labor profile choice of women born in period $t$ with the case where $w_t < w_{t+1}$, then we turn to the case where $w_t \geq w_{t+1}$.
3.3.1 The case where $w_{t+1} > w_t$

**Proposition 1**

If $w_{t+1} > w_t$, working in the first period in the market and in the second period at home (profile 3) cannot be optimal for any woman.

**Proof**

If $V(3) > V(1)$ it follows that $aw_t > H$. But since $w_{t+1} > w_t$ and $\theta > 1$ then $a\theta w_{t+1} > H$ which means that $V(4) > V(3)$. Therefore profile 3 cannot be optimal for any woman. ■

The rationale for this result is that if a woman had already worked in her life’s first period in the market, she accumulated market skills and since wages next period are higher next period then she must earn more in the market than at home in her life’s second period.

**Proposition 2:**

a. If $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$ then $a_{21,t} < a_{41,t} < a_{42,t}$ and the young women population is divided into three groups: young women with $a < a_{21,t}$ choose profile 1, young women with $a_{21,t} < a < a_{41,t}$ choose profile 2 and young women with $a > a_{42,t}$ choose profile 4.

b. If $w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < w_{t+1}$ then $a_{42,t} < a_{41,t} < a_{21,t}$ and the young women population is divided into two groups: young women with $a < a_{41,t}$ choose profile 1 and young women with $a > a_{41,t}$ choose profile 4.

**Proof**

See the appendix.

The results of proposition 2 are summarized in the following two sketches.
If $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$

The rationale behind the result of proposition 2 is as follows: When $w_t$ is sufficiently smaller than $w_{t+1}$, namely when $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$, the growth rate of wages is high. We therefore observe the ‘middle’ group: women who find that the loss in income in the first period is greater than the gain in income resulting from acquiring experience. However, since next period wage will rise significantly, it would be better to work next period in the market than at home. When $w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < w_{t+1}$ wages change relatively little over time. Thus if staying at home in the first period is optimal then staying at home in the second period is also optimal.

From the analysis above we can obtain the proportion of women, born in period $t$, who work in the modern sector in that period for the case where $w_{t+1} \geq w_t$:
3.3.2 The case where \( w_t \geq w_{t+1} \)

**Proposition 3**

If \( w_t \geq w_{t+1} \) working in the first period at home and in the second period in the market (profile 2) cannot be optimal for any woman.

**Proof**

If \( V(2) > V(4) \) it follows that \( H > aw_t \). But since \( w_{t+1} < w_t \) then \( aw_{t+1} < H \) which mean that \( V(1) > V(2) \). Therefore, profile 2 cannot be optimal for any woman. \( \blacksquare \)

**Proposition 4:**

a. If \( w_{t+1} < w_t < \theta w_{t+1} \) then \( a_{43,t} < a_{41,t} < a_{31,t} \). Young women with \( a < a_{41,t} \) choose profile 1 and young women with \( a > a_{41,t} \) choose profile 4.

b. If \( w_t > \theta w_{t+1} \), then \( a_{31,t} < a_{41,t} < a_{43,t} \). Young women with \( a < a_{31,t} \) choose profile 1, young women with \( a_{31,t} < a < a_{43,t} \) choose profile 3 and young women with \( a > a_{43,t} \) choose profile 4.

**Proof**

See the appendix.

From the above analysis we can obtain the proportion of women, born in period \( t \), who work in the modern sector in that period for the case where \( w_t \geq w_{t+1} \):

\[
(11) \quad x_t^A = \begin{cases} 
1 - a_{42,t} & \text{if } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \\
1 - a_{41,t} & \text{if } w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < w_{t+1} 
\end{cases}
\]
Combining both cases \((w_t < w_{t+1} \text{ and } w_t \geq w_{t+1})\) we get:

\[
x_t^A = \begin{cases} 
1 - a_{31,t} & \text{if } w_t > \theta w_{t+1} \\
1 - a_{41,t} & \text{if } w_{t+1} < w_t < \theta w_{t+1} 
\end{cases}
\]

The first line of (13) describes the case where some women born in period \(t\) find it optimal to join the labor market only in period \(t+1\). This occurs when the return for these women from joining the labor market already in period \(t\) is sufficiently low. Technically, it occurs when the period \(t\) wage and the experience multiplier satisfy: \(w_t(\theta-1) < 1/16\).

4 Equilibrium and Dynamics

In each period \(t\) two stocks are created and transferred to period \(t+1\): the stock of physical capital, \(K_{t+1}\), and a stock of women in their life’s second period with modern sector work experience, \(x_t^A\). In this section we show that given the initial values of these stocks, denoted by \(K_0\) and \(x_0^A\), a unique set of \(\{L_t^A, L_t^B, L_t^C, L_t^D, w_t, R_t\}_{t=0}^\infty\) that clears the labor market and the capital market in each period exists.

\footnote{The appendix shows \(x_t^A(w_t, w_{t+1})\) explicitly.}
Unlike many dynamical macroeconomic models, it is not possible in our model to obtain a solution to period $t$ variables by solving a finite set of equations that depend on these period $t$ variables alone, given the stocks created in period $t-1$. Instead, given $K_t$ and $x^A_{t-1}$, period $t$ variables must be found jointly with the variables of periods $t+1$, $t+2$, ..., $\infty$. This is because the labor supply of women born in period $t$ depends not only on current wage, $w_t$, but also on future wage, $w_{t+1}$.

We develop this section in the following manner: first we show that $(x^A_t, w_{t+1})$ is uniquely determined by $(x_t^A, w_t)$. Studying this dynamical system we show, using a numerical example, that it has a unique steady state with a unique saddle path leading to it. Then, we show that given the initial stocks, $K_0$ and $x^A_{-1}$, there is a unique point on this saddle that is consistent with markets clearing in period 0, and consequently in all subsequent periods too. Thus, $K_0$ and $x^A_{-1}$ uniquely determine the equilibrium $(x^A_t, w_t)_{t=0}^\infty$ and provide for each period $t$ the value of $w_{t+1}$ required for finding the period $t$ equilibrium values for the variables of the model.

### 4.1 The Dynamical System $(x^A_t, w_t)$

In this sub-section we show that $(x^A_{t+1}, w_{t+1})$ is uniquely determined by $(x^A_t, w_t)$. From (13) it follows that in the relevant range $w_{t+1}$ can be shown as a function of $x^A_t$ and $w_t$, we denote this function by $w_{t+1}(x^A_t, w_t)$ and present it explicitly in the appendix.

The stock of capital available in period $t+1$ is also a function of $x^A_t$ and $w_t$. The stock of capital available in period $t+1$ equals the savings done in period $t$. These savings are the incomes of the generation born in period $t$:

$$K_{t+1}(x^A_t, w_t) = I^A_t w_t + I^C_t w_t + (1 - x^A_t)H + (1 - x^C_t)P$$
Note that $x_t^C$ and $L_t^C$ are functions of $w_t$, as follows from (8) and (9). $L_t^A$ is a function of $x_t^A$, as follows from (5).

Manipulating (2) yields $L_{t+1}$ as a function of $w_{t+1}$ and $K_{t+1}$ and therefore as the following function of $x_t^A$ and $w_t$:

\[
L_{t+1}(x_t^A, w_t) = \frac{K_{t+1}(x_t^A, w_t)}{4w_{t+1}(x_t^A, w_t)}
\]

By definition $L_{t+1}^z$, where $z \in \{A, B, C, D\}$, satisfy $L_{t+1}^A = L_{t+1}^A + L_{t+1}^B + L_{t+1}^C + L_{t+1}^D$. Note from (9) and (10) that $L_{t+1}^C$ and $L_{t+1}^D$ are functions of $w_{t+1}$ and therefore of $x_t^A$ and $w_t$. $L_{t+1}^B$, the period $t+1$ labor supply of women born in period $t$, is a function of $x_t^A$ and $w_{t+1}$, as follows from section 3.2. Thus, in labor market equilibrium $L_{t+1}^A$ is the following function of $x_t^A$ and $w_t$:

\[
L_{t+1}^A(x_t^A, w_t) =
\]

\[
L_{t+1}(x_t^A, w_t) - L_{t+1}^B[x_t^A, w_{t+1}(x_t^A, w_t)] - L_{t+1}^C[w_{t+1}(x_t^A, w_t)] - L_{t+1}^D[w_{t+1}(x_t^A, w_t)]
\]

Finally, applying (16) into (5), and manipulating it yields $x_{t+1}^A$ as a function of $x_t^A$ and $w_t$:

\[
x_{t+1}^A(x_t^A, w_t) = 1 - \sqrt{1 - 2L_{t+1}^A(x_t^A, w_t)}
\]

\[\text{Equation (A.1) in the appendix shows this function explicitly.}\]
Thus, the model generates a dynamical system where $x_{t+1}^A$ and $w_{t+1}$ are uniquely determined by $x_t^A$ and $w_t$. Equations (A.4) and (A.5) in the appendix show this system explicitly.

Define the $ww$ curve as the pairs of $\left(x_t^A, w_t\right)$ for which the dynamical system yields $w_{t+1}=w_t$. Likewise, define the $xx$ curve as the pairs of $\left(x_t^A, w_t\right)$ for which this system yields $x_{t+1}^A=x_t^A$. Differentiating $w_{t+1}\left(x_t^A, w_t\right)$ with respect to $x_t$, yields $\frac{\partial w_{t+1}}{\partial x_t}>0$ which implies that above the $ww$ curve $w$ increases and vice versa. Similarly, differentiating $x_{t+1}^A\left(x_t^A, w_t\right)$ with respect to $w_t$ yields $\frac{\partial x_{t+1}^A}{\partial w_t}>0$ which implies that above the $xx$ curve $x$ decreases and vice versa.

Figure 1 shows the phase diagram of this dynamical system given the following parameters values: $P=0.025$, $H=0.02$ and $\theta=1.5$. As the figure shows, $ww$ and $xx$ cross each other only once, implying the existence of a unique steady state equilibrium. Given these parameters, one of the eigenvalues of the dynamical system linearized around the steady state is between 0 and 1, and the other eigenvalue is smaller than $-1$. This implies that the steady state is a saddle.6

4.2 Determination of the Equilibrium

In this sub-section we show that given the initial stocks, $K_0$ and $x_{-1}^A$, there is a unique point on the saddle path of the $\left(x_t^A, w_t\right)$ system that is consistent with markets clearing in period 0, and consequently in all subsequent periods too.

Given $K_0$ and $x_{-1}$, the following condition, based on (15) and (16), must hold in period 0 equilibrium:

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6 The derivation of the general equations for the $w$ and $x^A$ in the steady state and for the eigenvalues of the system appears in a mathematical appendix available from authors.
Since by (5) \( L_0^A \) is an increasing function of \( x_0^A \), (18) shows a negative relation between the values of \( w_0 \) and \( x_0^A \) that clear the labor market in period 0, given \( K_0 \) and \( x_{-1}^A \). This is because the demand for labor in the modern sector, \( L_0(K_0, w_0) \), is a decreasing function of \( w_0 \), while the labor supply for the modern sector by groups B, C and D is an increasing function of \( w_0 \).

Using the values of \( P, H \) and \( \theta \) from the previous sub-section and also the values \( K_0 = 0.0001 \) and \( x_{-1}^A = 0.02 \), figure 1 also shows the \( xw \) curve. The increasing dashed line in this figure shows points on the saddle path leading to the steady state in the \( (x_t^A, w_t) \) system. The figure shows a single cross between the saddle path and the \( xw \) curve. This point, \( (x_0^A, w_0) \), is the only one that satisfies not only condition (18) but also labor market and capital market clearing for each period \( t, t \geq 0 \). Thus, the initial stocks, \( K_0 \) and \( x_{-1}^A \), uniquely determine the dynamics of the economy since they determine a unique set \( \{ (x_t^A, w_t) \}_{t=0}^{\infty} \) consistent with market clearing over time.

### 4.3 An Application

In this subsection we show the possibility of non-monotonic dynamics of the ratio of the mean wage of working women to that of men. We denote this ratio by \( rw_t \).

Given the parameter values of the previous section, if the initial conditions, \( K_0 \) and \( x_{-1} \), are sufficiently low, the saddle path crosses the \( xw \) curve in a range where each of the dynamic labor profiles 1, 2, and 4, are optimal for some young women. The least able women choose profile 1, which means that they stay at home for their two life’s period; the more able women choose profile 2 which means that they stay at home in their life’s
first period and work in the modern sector in their life’s second period; the most able women choose profile 4, which means that they work in the modern sector in both their life’s period. Thus, since in that range profile 2 is optimal for some women, there are men and women in their life’s second period who are newcomers to the modern sector. In the previous period, these men acquired market skills by working in the physical sector, whereas these women worked at home and therefore have no market skills. This causes the relative market skills of women (to men) in the modern sector to decrease and thus the ratio of the mean wage of working women to the mean wage of working men decreases. As the economy develops the growth in wages slows down until. When the rate of growth of wages is sufficiently low profile 2 is no longer optimal for any woman. Thus, at this stage all the workers in their life’s second period are endowed with market skills and the reason for the decrease in rw is removed. The increase in wages in the modern sector affects women average income more than it affects men’s, since female LFP consists only of work in the modern sector while men also work in the physical sector. As a result, in this stage rw increases. Figure 2a shows this non-monotonic dynamics of rw, given the above values of P, H, θ, K0 and x1. Figure 2b shows the reason why profile 2 existed in the early stage of the development of the economy but vanished later - the concavity of the increase in wages in the modern sector.

5. Gender Gap Data and the Dynamics in the Model

In this section we compare the dynamics of the gender gap implied by the model with the empirical data. Smith and Ward (1989) show that the gender gap in 1980 was larger than in 1920. They also show that between 1968 and 1986 the gender gap has been gradually decreasing. Such a non-monotonic dynamics were also documented by O’Neil and Polacheek (1993). While Smith and Ward (1989) estimate the ratio between the mean income of women to that of men, O’Neil and Polacheek (1993) look at ratio of
median earnings. However, in years that appear in both studies the difference between the two ratios is negligible. Therefore we combine the data from both sources and present it in figure 3. The figure shows that the ratio of mean incomes has presented a W-shaped dynamics with a local maximum in the 1950’s.

It is possible to interpret the local peak in the 1950’s as a result of a temporary decrease in the value of home production during World War II. Goldin (1991, p.741) states that: “A husband’s absence often meant that his wife has less to do in the home”. Obviously, this decline in the value of home production can be viewed as a temporary shock. Goldin (1991) reports that 25 percent of the working women in the group age of 27-51 in 1951 were women who did not work in December 1941 but worked in March 1944 and January 1951. According to the composition effect described in our model the substantial growth in female LFP during the 1940’s would lead to a similarly sharp decline in the experience differences between men and women during the 1950’s.

Following this logic, inserting an exogenous temporary decrease to the value of home production in our model \( (H) \) would lead to a W-shaped dynamics instead of the U-shaped dynamics derived in the previous sections.

6. Concluding Remarks

In this paper we have developed a model that generates U-shaped dynamics of the ratio of the mean wages of women to that of men, as observed in the US during the 20th century. Our approach follows the empirical studies that have shown that when this ratio was decreasing many women were entering the labor force in a late phase of their life with low skills and little labor market experience, and that this ratio started increasing together with the disappearance of this behavior of women. In our model entering the

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8 Assuming that the temporary decline in the value of home production during the war time was followed by a temporary increase (relative to the value before the war) would only strengthen this argument. Such an increase could be attributed to the “baby boom” after the war.
labor market in a relatively late phase of their lives is optimal for some women when wages grow sufficiently rapidly but later disappears when wages growth slows down.

In the model we assume that individuals have perfect foresight. Assuming alternatively that individuals are myopic in the sense that their choice whether to work at home or in the market in a certain period depends only on the wages in that period, would not change the main qualitative results of the paper. This occurs because these results rely on the existence of a group of women who choose a labor profile in which they work at home in that period and work in the market in the second period, if wages grow sufficiently fast. Each woman that chooses such profile taking into account the growth in wages and the role of market experience would necessarily do so if in each period she were merely comparing the earnings from working at home to her earnings in the market in that period. In fact, this group is even larger under myopic behavior since under perfect foresight there are women who work in the market in the first period of their lives although they earn in that period less than they could earn at home. If these women were myopic they would not work in the market in the first period of their lives. Technically, the model under the assumption of myopic behavior would be much simpler since equilibrium in each period would not depend on the future equilibrium.

Another strong assumption in our model, taken for simplicity sake, is that productivity at home is constant. Greenwood, Seshardi and Yorukoglu (2001) have shown that timesaving technological improvements in home production play an important role in explaining female LFP dynamics in the past century in the US. Incorporating such progress in home production in our model requires that the assumption that individuals either works at home or in the market in each period should be replaced by the assumption that in each period individuals optimally allocate part of their time to home production and the rest to LFP. Such technological improvements in home production should have the same positive effect on individuals LFP decision as an increase in wages. Thus our conclusion that the gender gap decreases when wages grow sufficiently fast and vice versa, would be modified and become that the gender gap
decreases when the growth in wages, or the time saving technological progress in home production, are sufficiently fast, and vice versa.
Appendix

A. The function $L^b_{t+1}(x_t, w_{t+1})$

The following equation summarizes the results of section 3.2

\[
L^b_{t+1}(x_t, w_{t+1}) = \begin{cases} 
1 - \left( \frac{H}{\theta w_{t+1}} \right)^2 & \text{if} \quad a^x_t < \frac{H}{\theta w_{t+1}} \\
\frac{1 - (1 - x_t)^2}{2} & \text{if} \quad \frac{H}{\theta w_{t+1}} < a^x_t < \frac{H}{w_{t+1}} \\
\theta - (\theta - 1)(1 - x_t)^2 - \left( \frac{H}{w_{t+1}} \right)^2 & \text{if} \quad \frac{H}{w_{t+1}} < a^x_t 
\end{cases}
\]

(A.1)

B. The ability thresholds

Given the definition of $a_{mn,t}$ and the calculation of $V(i)$ the following holds:

\[
a_{21,t} = \frac{H}{w_{t+1}}, \quad a_{31,t} = \frac{H}{w_t}, \quad a_{41,t} = \frac{H + \frac{H}{R_{t+1}}}{w_t + \frac{\theta w_{t+1}}{R_{t+1}}} = \frac{H + 4hw_{t+1}}{w_t + 4\theta w_{t+1}^2},
\]

(A.2)

\[
a_{43,t} = \frac{H}{\theta w_{t+1}}, \quad a_{42,t} = \frac{H}{w_t + \frac{(\theta - 1)w_{t+1}}{R_{t+1}}} = \frac{H}{w_t + (\theta - 1)w_{t+1}^2},
\]

where the second equalities in $a_{42,t}$ and $a_{41,t}$ are obtained by applying $R_{t+1}$ from (4).

C. Proof of propositions 2 and 4

Regarding part (a) of proposition 2: The result that if $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$ then $a_{21,t} < a_{41,t} < a_{42,t}$ follows directly from (A.2). The division of the young women
population to three groups follows from the definition of these thresholds. The proof of part (b) of proposition 2 and the proof of proposition 4 are similar.

D. The function \( x_t^A(w_t, w_{t+1}) \)

Substituting (A.2) into (13) yields:

\[
\begin{align*}
  x_t^A &= \begin{cases} 
    1 - \frac{H}{w_t + (\theta - 1)4w_{t+1}} & \text{if } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \\
    1 - \frac{H(1 + 4w_{t+1})}{w_t + 4\theta w_{t+1}} & \text{if } w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < \theta w_{t+1} \\
    1 - \frac{H}{w_t} & \text{if } \theta w_{t+1} < w_t
  \end{cases}
\end{align*}
\]  

(A.3)

Note that despite the division to three ranges, \( x_t^A \) is continuous in \( w_t \) and \( w_{t+1} \).

Differentiation of the first two lines of (A.3) shows that \( \frac{\partial x_t^A}{\partial w_{t+1}} > 0 \) and therefore that

\[ x_t^A \geq 1 - \frac{H}{w_t} \text{ for all } w_{t+1}. \]

E. The function \( w_{t+1}(x_t^A, w_t) \)

For convenience of notation we define \( h_t(x_t^A) \equiv h_t \equiv \frac{H}{1 - x_t^A} \). Since \( x_t^A \geq 1 - \frac{H}{w_t} \) for all \( w_{t+1} \) it follows that \( h_t \geq w_t \).

Manipulating the first line in (A.3) in order to isolate \( w_{t+1} \) yields \( w_{t+1} = \frac{h_t - w_t}{\sqrt{4(\theta - 1)}} \).

Applying this in \( w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \) shows that this case is relevant in the range
\[ E \equiv \left\{ (w_t, x_t^A) : w_t < h_t - 4(\theta - 1)h_t^2 \right\}. \]
Likewise, isolating \( w_{t+1} \) in the second line of (A.3) yields
\[ w_{t+1} = \frac{h_t + \sqrt{h_t^2 - \theta(w_t - h_t)}}{2\theta} \]
and the relevant range becomes
\[ F \equiv \left\{ (w_t, x_t^A) : h_t - 4(\theta - 1)h_t^2 < w_t < h_t \right\}. \]
In the third line of (A.3) \( w_t > \theta w_{t+1} \) implying a rapid decline in wages over time. The economy will therefore be in this range only if the initial stock of physical capital is sufficiently large. Note that in this range there is a set of values of \( w_{t+1} \), rather than a single value, that corresponds to a given pair of \( (w_t, x_t^A) \).
Since the case of rapidly declining wages is not in the focus of this paper, we assume that the initial stock of physical capital is not that large. Summarizing this analysis shows that:

\[
\begin{cases}
\frac{h_t - w_t}{4(\theta - 1)} & \text{if } (w_t, x_t^A) \in E \\
\frac{h_t + \sqrt{h_t^2 - \theta(w_t - h_t)}}{2\theta} & \text{if } (w_t, x_t^A) \in F
\end{cases}
\]

Note that \( x_t^A \) appears on the RHS of (A.4) through \( h_t \).

\textbf{F. The function } x_{t+1}^A = (x_t^A, w_t) \textbf{ }

Following the procedure presented in detail in section 3.1 the explicit form of the function \( x_{t+1}^A = (x_t^A, w_t) \) is:

\textit{In the range where } \( (w_t, x_t^A) \in E \text{ and } w_t \geq P \)
In the range where \((w_i, x_i^A) \in E\) and \(w_i < P\)

\[
x_{i+1}^A = 1 - \sqrt[2]{2(\theta + 1) - 2(\theta - 1) + \frac{w_i}{h_i} - \frac{(H - h_i)^2}{2w_i h_i} + \frac{P^2}{h_i} + \frac{H^2}{h_i} + 2(1 + \theta)P^2 + 2H^2\} - (\theta - 1)\left(\frac{H}{h_i}\right)^2
\]

In the range where \((w_i, x_i^A) \in F\) and \(w_i \geq P\)

\[
x_{i+1}^A = 1 - \sqrt[2]{-2(\theta - 1) + \frac{1 - \left(\frac{H}{h_i}\right)^2}{2} + \frac{H^2}{h_i} + P + 2H^2\} + (\theta + 1) - (\theta - 1)\left(\frac{H}{h_i}\right)^2
\]

In the range where \((w_i, x_i^A) \in F\) and \(w_i < P\)

\[
x_{i+1}^A = 1 - \sqrt[2]{2(\theta + 1) - 2\theta^2 - \frac{w_i}{2} - \frac{(H - h_i)^2}{2w_i h_i} + \frac{P^2}{h_i} + \frac{H^2}{h_i} + 2(\theta + 1)P^2\} - \theta\left(\frac{H}{h_i}\right)^2
\]

In the range where \((w_i, x_i^A) \in F\) and \(w_i < P\)

\[
x_{i+1}^A = 1 - \sqrt[2]{-2\theta^2 - \frac{w_i}{2} - \frac{(H - h_i)^2}{2w_i h_i} + \frac{P^2}{h_i} + \frac{H^2}{h_i} - \theta\left(\frac{H}{h_i}\right)^2 + \theta + 1}
\]
References


Parameter values: $H=0.02$, $P=0.025$, $\theta=1.5$, $K_0=0.0001$, $x_{-1}^A=0.02$
Figure 2.a - The ratio of means wages over time

Parameter values: $H=0.02$, $P=0.025$, $\theta=1.5$, $K_0=0.0001$, $x_{-1}^A=0.02$

Figure 2.b - The wage for one unit of efficiency labor over time

Parameter values: $H=0.02$, $P=0.025$, $\theta=1.5$, $K_0=0.0001$, $x_{-1}^A=0.02$
Figure 3: The ratio of mean earnings in the US*

See text for details.