Speaker: Pieter Allart, University of North Texas

Title: Prophet inequalities for i.i.d. random variables with random arrival times

Abstract: Suppose $X_1, X_2, \ldots$ are i.i.d. nonnegative random variables with finite expectation, and for each k, X_k is observed at the k-th arrival time S_k of a Poisson process with unit rate which is independent of the sequence $\{X_k\}$. Let $t > 0$ be a finite time horizon. Several comparisons are given between the expected maximum

$$M(t) := E\left[\max_{k \geq 1} X_k I(S_k \leq t)\right]$$

and the optimal stopping value

$$V(t) := \sup_{\tau} E[X_\tau I(S_\tau \leq t)],$$

where the supremum is taken over all N-valued random variables τ such that $\{\tau = i\}$ is measurable with respect to the σ-algebra generated by $X_1, \ldots, X_i$ and $S_1, \ldots, S_i$. For example, $M(t)/V(t) \leq 1 + \alpha_0$, where $\alpha_0 \doteq 0.34149$ is the unique value of α such that $\int_0^1 (y - y \ln y + \alpha)^{-1}\, dy = 1$. This bound is asymptotically sharp as $t \to \infty$. Another result is that $M(t)/V(t) < 2 - (1 - e^{-t})/t$, and this bound is asymptotically sharp as $t \downarrow 0$. Analogous upper bounds for the difference $M(t) - V(t)$ are also given, under the additional assumption that the X_k are bounded. The relationship between these 'prophet inequalities' and classical prophet inequalities for sequences of i.i.d. random variables by Hill and Kertz will be discussed.