UNITED STATISTICS: PARAMETER CONFIDENCE QUANTILES, DUALITY BAYESIAN FREQUENTIST INFERENCE

by Emanuel Parzen, Statistics Department, Texas A&M University

A quantile function $Q(P), 0 < P < 1$, is inverse $F^{-1}(P)$ of distribution function $P = F(Q), -\infty < Q < \infty$. We regard statistical inference as inverse probability, quantile functions can provide universal algorithms for data modeling. Quantile exploratory data analysis of data $Y_1, \ldots, Y_n$, computes sample quantile function $Q(P; Y, \text{sample})$, identifies probability models from

$$QIQ(P) = \big(Q(P; Y, \text{ sample}) - MQ\big)/SQ, MQ = .5(Q1 + Q3), SQ = 2(Q3 - Q1).$$

Parametric modeling of data $Y_1, \ldots, Y_n$ (1) assumes a parametric model $f(Y|\theta)$, (2) computes maximum likelihood estimator $\hat{\theta}$, (3) computes for each θ probability $P = PVALUE(\theta, \hat{\theta}_{obs}) = Pr_\theta[\hat{\theta} \geq \hat{\theta}_{obs}]$, (4) assume continuous increasing function of θ, (5) compute inverse PVALUE or confidence quantile $Q(P; \theta|\hat{\theta} = \hat{\theta}_{obs}, \text{ conf})$. For Y Normal (θ, σ^2), σ known, confidence quantile equals $\bar{Y} + (\sigma/\sqrt{n})\Phi^{-1}(P)$. Values of parameter θ for which $\hat{\theta}_{obs}$ is probable is defined for specified confidence level $1 - \alpha$ to be confidence interval $\alpha/2 \leq PVALUE(\theta, \hat{\theta}_{obs}) \leq 1 - (\alpha/2)$, equivalently $Q(\alpha/2; \theta|\hat{\theta}_{obs},$ conf)$\leq \theta \leq Q(1 - (\alpha/2); \theta|\hat{\theta}_{obs}, \text{ conf})$. Bayesian credible interval endpoints are computed from posterior quantile $Q(P; \theta|$ data, prior). Plot on (P, θ) plane prior quantile, posterior quantile, confidence quantile, relative likelihood $P = f(Y_1, \ldots, Y_n|\theta)/f(Y_1, \ldots, Y_n|\hat{\theta}_{obs})$, and confidence intervals they determine. For n Berrnouli trials with k successes plot on (P, p), plane prior quantile Beta (a, b), posterior quantile Beta$(k + an - k + b)$, confidence quantile by Wilsons formula, relative likelihood $P = \exp$ (-Information Divergence $(\hat{p}, p)$). Application to Karl Pearson (1920) Fundamental Problem of Practical Statistics.